Parameterized approximability of maximizing the spread of influence in networks

Cristina Bazgan, Morgan Chopin, André Nichterlein, Florian Sikora

PII: S1570-8667(14)00031-8
DOI: 10.1016/j.jda.2014.05.001
Reference: JDA 555

To appear in: *Journal of Discrete Algorithms*

Received date: 26 July 2013
Revised date: 14 March 2014
Accepted date: 28 May 2014

Parameterized Approximability of Maximizing the Spread of Influence in Networks☆

Cristina Bazgan[a,d], Morgan Chopin[b], André Nichterlein[c], Florian Sikora[a]

[a] *PSL, Université Paris-Dauphine, LAMSADE UMR 7243, France.*
`{bazgan,florian.sikora}@lamsade.dauphine.fr`
[b] *Institut für Optimierung und Operations Research, Universität Ulm, Germany.*
`morgan.chopin@uni-ulm.de`
[c] *Institut für Softwaretechnik und Theoretische Informatik, TU Berlin, Germany.*
`andre.nichterlein@tu-berlin.de`
[d] *Institut Universitaire de France*

Abstract

In this paper, we consider the problem of maximizing the spread of influence through a social network. Given a graph with a threshold value $\mathrm{thr}(v)$ attached to each vertex v, the spread of influence is modeled as follows: A vertex v becomes "active" (influenced) if at least $\mathrm{thr}(v)$ of its neighbors are active. In the corresponding optimization problem the objective is then to find a fixed number k of vertices to activate such that the number of activated vertices at the end of the propagation process is maximum. We show that this problem is strongly inapproximable in time $f(k) \cdot n^{O(1)}$, for some function f, even for very restrictive thresholds. In the case that the threshold of each vertex equals its degree, we prove that the problem is inapproximable in polynomial time and it becomes $r(n)$-approximable in time $f(k) \cdot n^{O(1)}$, for some function f, for any strictly increasing function r. Moreover, we show that the decision version parameterized by k is W[1]-hard but becomes fixed-parameter tractable on bounded degree graphs.

Keywords: Parameterized Complexity, Approximation, Parameterized Approximation, Target Set Selection, Dynamic Monopolies, Spread of Information, Viral Marketing

1. Introduction

Optimization problems that involve a diffusion process in a graph are well studied [23, 17, 10, 1, 14, 9, 4, 24, 3]. Such problems share the common property that, according to a specified *propagation rule*, a chosen subset of vertices activates all or a fixed fraction of the vertices, where initially all but the chosen vertices are inactive. Such optimization problems model the spread of influence or information in social networks via word-of-mouth recommendations, of diseases in populations, or of faults in distributed computing [23, 17, 14]. One representative problem that appears in this context is the *influence maximization* problem introduced by Kempe *et al.* [17]. Given a directed graph, the task is to choose a fixed number of vertices such that the number of activated vertices at the end of the propagation process is maximized.

☆An extended abstract appeared in the Proceedings of the 9th Annual International Computing and Combinatorics Conference (COCOON'13), volume 7936 of *LNCS*, pages 543-554. Springer, 2013.