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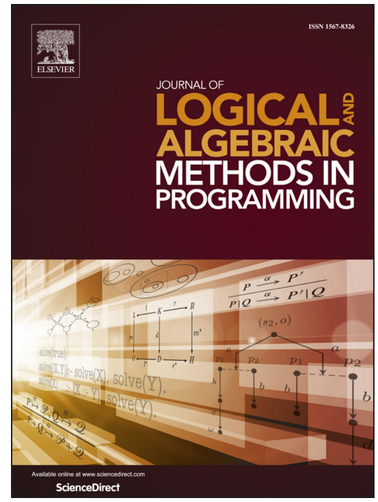

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Abstract

We characterize the intrinsically recursive functions over the algebraic closure of a finite field in terms of Turing machine complexity classes and derive some structural properties about the family of such functions. In particular, we show that the domain of convergence of any partial recursive function is again recursive, and, under complexity-theoretic hypotheses, that the class of tail recursive functions is strictly smaller than the class of recursive functions (cf. Theorems 5.4, 5.2, and Section 8).

Underlying these results is the "meta-result" that we can perform a limited amount of arithmetic inside the field itself, with no access to a separate sort of natural numbers.

Keywords: Generalized recursion theory, Finite fields, Tail recursion

1 Introduction

The classical notion of computability over the natural numbers can be extended in several ways to general first-order structures. Broadly speaking, these come in two sorts. The *extrinsic* approach involves encoding a structure $\mathbf{A}$ by natural numbers, and defining a function to be computable when its corresponding function of codes is computable in the ordinary sense. The *intrinsic* approach considers "generalized programs," syntactic objects written in the language of $\mathbf{A}$, which define relations and functions directly. In this paper, we are concerned with the latter.

Our standard point of reference is the "classical" recursion theory of the structure of *unary arithmetic,*

$$\mathbf{N}_u = (\mathbb{N}; 0, 1, S, Pd, eq_0),$$

where $eq_0(n) \iff n = 0$. To get a handle on the recursion theory of an arbitrary structure $\mathbf{A}$, we might naturally ask to what extent it resembles recursion theory over $\mathbf{N}_u$. Classically, there are recursive pairing and unpairing functions, a Gödel numbering that admits universal functions and s-m-n functions, and a

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