Doctopic: Algorithms, automata, complexity and games ARTICLE IN PRESS TCS:11577

Theoretical Computer Science ••• (••••) •••–•••

Contents lists available at ScienceDirect

Theoretical Computer Science

www.elsevier.com/locate/tcs

On time complexity for connectivity-preserving scattering of mobile robots ☆

Taisuke Izumi [a,*], Daichi Kaino [a], Maria Gradinariu Potop-Butucaru [b], Sébastien Tixeuil [b]

[a] *Graduate School of Engineering, Nagoya Institute of Technology, Japan*
[b] *Université Pierre et Marie Curie – Paris 6, LIP6 CNRS 7606, France*

A R T I C L E I N F O

Article history:
Received 18 May 2016
Received in revised form 28 October 2017
Accepted 24 April 2018
Available online xxxx
Communicated by L.A. Gasieniec

Keywords:
Autonomous mobile robots
Scattering problem
Time complexity

A B S T R A C T

The *scattering* problem is a fundamental task for mobile robots, which requires that no two robots share the same position. We investigate the scattering problem in the limited-visibility model. In particular, we focus on the *connectivity-preservation* property, *i.e.*, the scattering must be achieved so that the disconnection of the visibility graph never occurs (in the visibility graph nodes are robots and edges are their visibility relationship). This paper shows a new scattering algorithm in the semi-synchronous model, which guarantees the connectivity preservation property, and reaches a scattered configuration within $O(\min\{n, D^2 + \log n\})$ asynchronous rounds in expectation, where n is the number of robots and D is the diameter of the initial visibility graph. Note that a part of this complexity analysis is *adaptive* in the sense that it depends on D. This implies that our algorithm quickly scatters all robots crowded with a small-diameter visibility graph. We also show two matching lower bounds for the connectivity-preserving scattering problem. The first lower bound is $\Omega(n)$ rounds in the case of $D = \Omega(n)$, which is a lower bound applying to any connectivity-preserving algorithm. The second bound applies to a (reasonably) restricted class of algorithms, called *conservative algorithms*, which provides $\Omega(D^2)$-round lower bound. These two bounds imply that our algorithm achieves the optimal running time (in the sense of both adaptive and non-adaptive analyses) as conservative ones because it is conservative.

1. Introduction

Background. Algorithmic studies about autonomous mobile robots recently emerged in the distributed computing community. In most of those studies, a robot is modelled as a point in a Euclidean plane, and its abilities are quite limited: It is usually assumed that robots are *oblivious* (*i.e.* no memory is used to record past situations), *anonymous* (*i.e.* no ID is available to distinguish two robots), and *uniform* (*i.e.* all robots run the same identical algorithm). It is also assumed that each robot

☆ This work is supported in part by KAKENHI No. 15H00852 and 25289227. Preliminary versions of this paper appear at: D. Kaino and T. Izumi "On the Worst-Case Initial Configuration for Conservative Connectivity Preservation", SRDS Workshops, pp. 60–63, 2014. T. Izumi, M. Gradinariu Potop-Butucaru, S. Tixeuil "Connectivity-Preserving Scattering of Mobile Robots with Limited Visibility" SSS, pp. 313–331, 2010.

* Corresponding author.
E-mail address: t-izumi@nitech.ac.jp (T. Izumi).

https://doi.org/10.1016/j.tcs.2018.04.047
0304-3975/© 2018 Elsevier B.V. All rights reserved.

has no direct means of communication. The communication between two robots is done in an implicit way by having each robot observe its environment, which includes the positions of other robots.

A more challenging setting of the algorithmic robotics is the *limited visibility* model [1–3], where each robot can see only the robots within the unit visibility range (*a.k.a.* the unit distance range). The limited visibility is a practical assumption but makes the design of algorithms quite difficult because it prevents each robot from obtaining the global information about all other robots. Furthermore, it also brings another design issue, called *connectivity preservation*: Oblivious robots cannot use the previous history of their execution. Hence, once some robot r_1 disappears from the visibility range of another robot r_2, r_2 can behave as if r_1 does not exist in the system and vice versa. Hence the cooperation between r_1 and r_2 becomes impossible. This phenomenon is formally described by using a *visibility graph*, which is the graph induced by robots (as nodes) and their visibility relationship (as edges). The requirement we have to guarantee in the limited visibility model is that any task or sub-task in an algorithm must be achieved in the manner that it preserves the connectivity of the visibility graph.[1]

We consider the *scattering* problem, which is a fundamental task in the mobile robotics, in the limited visibility model. In this task, starting from an arbitrary configuration (*i.e.* arbitrary initial locations for participating robots which may contain two or more same locations), eventually no two robots share the same position. The scattering algorithm can be considered as the dual problem of gathering (making all robots reach a common point), and closely related to the deployment or coverage problem, which is a stronger and practical variant of the scattering problem [7,10,11]. They require all robots placed into the locations mutually not so near (to cover some target region). Clearly solving the deployment problem requires the scattering, and they share the inherent difficulty and complexity. Particularly, any complexity lower bound for the scattering problem also applies to the deployment problem.

Because of the hardness of symmetry breaking, it is trivially impossible to construct deterministic scattering algorithms. That is, there is no deterministic way to separate two robots on the same position into different positions if both of them execute synchronously. Thus, algorithms for the scattering problem inherently exploit *randomization*.

Our contribution. The contribution of this paper is to propose a probabilistic scattering algorithm with the connectivity preservation property. We also give a complexity analysis of the proposed algorithm. Interestingly, the analysis is adaptive in the sense that it depends on the diameter D of the visibility graph in the initial configuration. Our algorithm achieves scattering within $O(\min\{n, D^2 + \log n\})$ asynchronous rounds in expectation. This implies that our algorithm quickly scatters all robots initially crowded within a small diameter visibility graph (*i.e.*, loss of connectivity due to robot movement is unlikely).

Another interesting point is that we show time lower bounds for the round complexity of the scattering problem in our model. Two results are presented on this line: The first one is that despite highly-concurrent behaviour of distributed mobile robots, any scattering algorithm can be as slow as $\Omega(n)$ rounds (where n is the number of robots) in the worst case to ensure the connectivity preservation. The second result applies to a (reasonably) restricted class of algorithms, called *conservative algorithms*, which guarantees that no link deletion of the visibility graph occurs in any execution. Obviously, the conservativeness property restricts the movement of robots causing the edge deletion of visibility graphs. As the upper-bound side, the second result also targets adaptive bounds, that is, the time bound depending on D. A naive lower bound obtained by the same argument as the first one is $\Omega(D)$ rounds, and thus the goal is to fill the gap between this and the corresponding upper bound ($O(D^2)$ rounds). The second result we show is that it is actually filled to the negative side: Any conservative scattering algorithm takes $\Omega(D^2)$ rounds in the worst case. These two results imply that our algorithm is optimal in the class of conservative algorithms. It should be noted that conservativeness property is not so special. While it might look too strong restriction, surprisingly, most of known algorithms for the limited visibility model is conservative! Hence we believe that identifying the complexity of conservative algorithm is so insightful for limited-visibility algorithms.

Related work. While the scattering problem is mentioned in the seminal paper that originated algorithmic for mobile robots [4], only a limited number of contributions considered this problem.

The initiating paper by Suzuki and Yamashita [4] introduced the scattering problem and proposed a deterministic algorithm under the assumption that *clones* do not exist. Two robots are considered to be clones of each other if they have the same local $x-y$ coordinate system and the same initial position, and they always become active simultaneously. In [5], the authors formalize the scattering problem, and propose a probabilistic algorithm based on the concept of Voronoi diagrams (this technique typically requires that robots are endowed with full system visibility at all times). They also show how the scattering can be used in solving the pattern formation problem. Furthermore, [6] investigates the time complexity of scattering, and exhibit a relation between the time complexity and the robots capability to detect the multiplicity (the number of robots sharing the same position).

Flocchini et al. consider a stronger variant of the scattering problem that requires all robots to reach different positions uniformly distributed over some discrete space [7]. This direction is also recently explored by Barrière et al. [8].

[1] Note that the disconnection does not necessarily imply the impossibility of tasks because we do not rule out the possibility that algorithms reconnect visibility graphs. However, disconnection is anyway a catastrophic situation, and thus we define the connectivity preservation property as the one guaranteeing that algorithms never bring disconnection during executions.

Please cite this article in press as: T. Izumi et al., On time complexity for connectivity-preserving scattering of mobile robots, Theoret. Comput. Sci. (2018), https://doi.org/10.1016/j.tcs.2018.04.047