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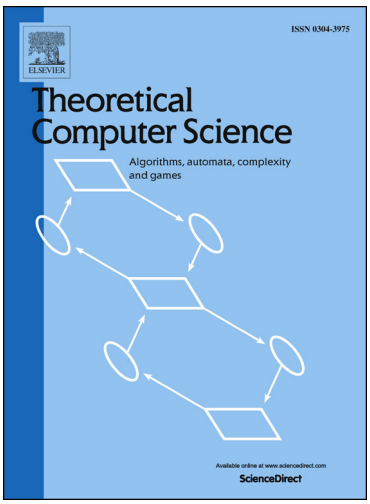

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On The Complexity of Bounded Time and Precision Reachability for Piecewise Affine Systems*

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Abstract. Reachability for piecewise affine systems is known to be undecidable, starting from dimension 2. In this paper we investigate the exact complexity of several decidable variants of reachability and control questions for piecewise affine systems. We show in particular that the region-to-region bounded time versions leads to NP-complete or co-NP-complete problems, starting from dimension 2. We also prove that a bounded precision version leads to *PSPACE*-complete problems.

1 Introduction

A (discrete time) dynamical system $\mathcal{H}$ is given by some space X and a function $f : X \to X$. A trajectory of the system starting from x_0 is a sequence $x_0, x_1, x_2, \ldots$ etc., with $x_{i+1} = f(x_i) = f^{[i+1]}(x_0)$ where $f^{[i]}$ stands for i^{th} iterate of f. A crucial problem in such systems is the *reachability question*: given a system $\mathcal{H}$ and $R_0, R \subseteq X$, determine if there is a trajectory starting from a point of R_0 that falls in R. Reachabilty is known to be *undecidable* for very simple functions f. Indeed, it is well-known that various types of dynamical systems, such as hybrid systems, piecewise affine systems, or saturated linear systems, can simulate Turing machines, see e.g., [12,9,14,15].

This question is at the heart of *verification* of systems. Indeed, a safety property corresponds to the determination if there is a trajectory starting from some set R_0 of possible initial states to the set R of bad states. The industrial and economical impact of having efficient computer tools, that are able to guarantee that a given system does satisfy its specification, have indeed generated very important literature. Particularly, many undecidability and complexity-theoretic results about the hardness of verification of safety properties have been obtained in the model checking community. However, as far as we know, the exact complexity of *natural restrictions* of the reachability question for systems as simple as piecewise affine maps are not known, despite their practical interest.

Indeed, existing results mainly focus on the frontier between decidability and undecidability. For example, it is known that reachability is undecidable

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