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Mechanism design for one-facility location game with obnoxious effects on a line ☆

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ABSTRACT

In this paper, we introduce obnoxious effects into obnoxious facility location games on a line, where each agent i has a private location x_i on a closed interval $[0, 1]$ and one facility will be built on a location y in the interval according to the bids of all the agents. In addition, there are two thresholds d_1 and d_2 in the utility function of each agent, where $0 \le d_1 \le d_2 \le 1$. Denote $d(y, x_i) = |y - x_i|$ to be the distance between agent i and the facility on the location y. The utility function of agent i is 0 if $d(y, x_i)$ is at most d_1; 1 if $d(y, x_i)$ is at least d_2; otherwise a linear increasing function between 0 and 1. Each agent attempts to get the largest utility while the social welfare is to maximize the sum of all the agents' utilities.

The classic obnoxious facility game [4,11] is a special case of our problem when $d_1 = 0$ and $d_2 = 1$. In this work, we first study the hardness of approximate mechanism design on this generalized problem, which states that our problem cannot admit any deterministic strategy-proof mechanism with bounded approximation ratio if $d_1 \ge \frac{1}{2}$. Then we limit the thresholds to some ranges, both deterministic and randomized strategy-proof mechanisms are studied, and the approximation ratios vary with the specific values of d_1 and d_2.

1. Introduction

Algorithmic mechanism design on various combinatorial optimization problems has been prevalently studied nowadays. Procaccia and Tennenholtz [14] first considered the approximate mechanism design without payment for facility location problems. A set of strategic agents have different locations, and a mechanism is expected to select a facility location based on the locations reported by all the agents. Each agent prefers to be as close to the facility as possible. Recently, mechanism design on obnoxious facility games was proposed by Cheng et al. [4], where each agent prefers to stay far away from the facility. Let us define each agent's utility (or cost) to be the distance to the facility in above mentioned facility location games, then the games can be classified into two groups. One is to maximize the utility and the other is to minimize the cost. In other words, in previous studies, the facilities may be classified to be either purely desirable where they should be close to the users, or purely undesirable which means they should be as far away as possible. This classification is not enough in practical scenarios, as being pointed out by Brimberg and Juel [2] that some types of facilities have obnoxious effects in the real life. For example, a government plans to build a garbage dump, a chemical plant, or a nuclear reactor

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on a street. When the facility is close to a resident within a fixed range, it is totally unacceptable to her. Similarly, if it is already far away enough, small increases in the distance cannot change the obnoxious effects.

In this work, taking the obnoxious effects into account, we consider mechanism design on the obnoxious facility game with two thresholds d_1 and d_2 which are two constants in $[0, 1]$ and $d_1 \le d_2$. Let $d(y, x_i)$ denote the distance between agent i and the facility that is located at y. The agent i's utility function $u_i(y, x_i)$ is defined as follows. The function $u_i(y, x_i)$ is 0 if $d(y, x_i)$ is at most d_1; 1 if $d(y, x_i)$ is at least d_2; otherwise a linear increasing function between 0 and 1 (the detailed function is given in Section 2). The social welfare is to maximize the sum of all the agents' utilities. We wish to design mechanisms without payment which must be strategy-proof, *i.e.*, preventing any agent from lying to benefit. At the same time, the proposed mechanisms are expected to have small approximation ratios with respect to optimal social welfares.

1.1. Related works

Mechanism design for facility location games has a considerable amount of work in the literature. To state conveniently, we use k-facility game to denote the facility location game where k facilities are built. Procaccia and Tennenholtz [14] first studied mechanism design for facility location games. In the setting, the cost of each agent is the distance from the facility to the agent's location and each agent attempts to minimize the cost. They considered two objective functions, minimizing the sum of all the agents costs(minSum) and the maximum cost(minMax). They gave some lower bounds and upper bounds for 1-facility game and 2-facility game. They also considered an extended model – multiple locations per agent. Subsequently, Alon et al. [1] extended the randomized and deterministic mechanisms for 1-facility game on other networks. Lu et al. [13,12] successively improved some results for 2-facility game on general metric networks under the minSum objective function. Feldman and Wilf [8] then considered the deterministic and randomized mechanisms for the objective function of minimizing the sum of squares of all the agents' utilities(minSOS). The number of facilities which is more than three was considered by Fotakis et al. [10] and Escoffier et al. [6]. Recently, Filos-Ratsikas et al. [9] considered a facility game where each agent prefers two locations named by double-peaked preferences.

Mechanism design for the obnoxious facility games was first studied by Cheng et al. [4]. They considered deterministic and randomized strategy-proof mechanisms for maxSum objective function on the line. Cheng et al. extended the results on the other networks in [5]. Ibara and Nagamochi [11] gave the characterization of strategy-proof mechanisms for the obnoxious facility game for the maxSum objective function. Subsequently, Cheng et al. [3] investigated the obnoxious facilities with a bounded service range. Recently, Ye et al. [16] gave some results for the objectives of maxSum and maxSOS. They also considered the extended model of multiple locations per agent for maxSum and maxSOS objectives.

Zhang and Li [17] studied the weighted version for both facility games and obnoxious facility games. Moreover, they considered the facility location game with one threshold with respect to the utility function, an optimal deterministic mechanism was provided. Zou and Li [18] considered the problem where two preferences of agents, staying close to and staying away from the facility, exist. They called this problem the facility location game with dual preference. They gave a deterministic mechanism. They also considered the two-opposite-facility location game with limited distance. Here, two-opposite-facility location means one facility is that each agent want to stay close to; the other one is opposite. Feigenbaum and Sethuraman [7] considered randomized mechanisms for the facility location game with dual preference.

On the optimization aspect, our work is related to approximation algorithms for semi-desirable k-facility location problem [2,15].

1.2. Our results

The optimization version of our problem generalizes the obnoxious facility on a line problem. We show that the optimization version of this problem still can be solved in polynomial time. However, it remains challenge for mechanism design on this problem since there is no payment function involved. We explore the problem with various combination of the two thresholds. When $d_1 = d_2$, *i.e.*, there is only one threshold in our problem, the problem becomes trivial since a mechanism that chooses the leftmost optimal facility location is strategy-proof. So, without loss of generality, we assume that $0 \le d_1 < d_2 \le 1$. We study the hardness of approximate mechanism design by showing that the approximation ratios of any deterministic strategy-proof mechanisms are unbounded when $d_1 \ge 1/2$. Next, we study randomized mechanisms for this case. On the positive side, we design a 2-approximate randomized strategy-proof mechanism. On the negative side, we show that no randomized mechanism can achieve an approximation ratio less than $4/3$ in expectation.

To address the case when $d_1 < 1/2$, we propose a majority mechanism motivated by Cheng et al. [4] and show its approximation ratio with respect to d_1 and d_2. Furthermore, we show that the mechanism is best possible for some d_1 and d_2. For those cases that the majority mechanism is not best possible, we provide a family of improved deterministic mechanisms, *i.e.*, the approximation ratio is $(1 + \frac{1}{k})$ for $d_1 < d_2 \le 1/2$ and k is an integer such that $\frac{1}{2(k+1)} < d_2 \le \frac{1}{2k}$.

The rest of this paper is organized as follows. In Section 2, we present preliminaries and detailed definition of the utility function. Section 3 provides an optimal algorithm for the optimization version of our problem and study the hardness of approximate mechanisms. Section 4 explores approximate mechanisms for many combinations of the two thresholds. Finally, concluding remarks are given in Section 5.

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