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Parameterized counting matching and packing: A family of hard problems that admit FPTRAS ☆

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ABSTRACT

In the field of parameterized counting complexity, the problems that are #W[1]-hard and admit fixed-parameter tractable randomized approximation scheme (FPTRAS) have attracted much attention in recent years. In this paper, we focus on the problems on parameterized counting matching and packing. These problems include COUNTING SET PACKING, COUNTING MATCHING, and COUNTING SUBGRAPH PACKING (including both vertex-disjoint and edge-disjoint versions). We study the parameterized complexity on these problems. On the basis of some results for counting graph matchings, we show that a series of problems are #W[1]-hard. Furthermore, by extending the previous algorithm for counting 3-D MATCHING, we obtain FPTRAS for each considered problem, respectively. Our results indicate that the problems on parameterized counting matching and packing form a large family of problems that are #W[1]-hard and admit FPTRAS.

1. Introduction

Counting the number of some special structures in a given model is one of fundamental computations in theoretical computer science. It also has a broad application in computational biology, statistics physics, and artificial intelligence. Parameterized counting complexity theory, a framework for studying the complexity of counting problems from a parameterized point of view, has been an important branch in computational complexity theory [14,22,33].

In the field of parameterized counting complexity, one main line of research is to investigate the complexity of counting problems and to explore the fixed-parameter tractable randomized approximation scheme (FPTRAS) for the intractable problems [2,9,16,20,23]. The problems that are #W[1]-hard and admit FPTRAS attract a lot of attention. However, for many counting problems, the results on two aspects have not been obtained completely. On one hand, for many #W[1]-hard problems, the FPTRAS have not been presented. Especially, some #W[1]-hard problems were shown not to admit FPTRAS under some assumptions [23]. On the other hand, for some problems that admit FPTRAS, the hardness results remain unknown [16].

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In this paper, we focus on the problems on parameterized counting matching and packing. These problems include COUNTING SET PACKING, COUNTING MATCHING, COUNTING VERTEX-DISJOINT SUBGRAPH PACKING, and COUNTING EDGE-DISJOINT SUBGRAPH PACKING. We aim to present a large family of problems that are #W[1]-hard and admit FPTRAS.

We first give some definitions about parameterized COUNTING SET PACKING. Let $\mathcal{S}$ be a collection of n sets, each of which contains at most m elements from a universal set U. A set is called an m-set if it contains exactly m elements. Let $P \subseteq \mathcal{S}$. P is a *packing* if any two sets in P do not intersect. The size of P is the number of sets in P. A packing is a k-*packing* if it contains exactly k sets. We consider two problems on parameterized COUNTING SET PACKING.

Definition 1.1. p_k-#m-SET PACKING: Given a pair $(\mathcal{S}, k)$, where $\max\{|s||s \in \mathcal{S}\} \le m$ and k is the parameter, count the number of distinct k-packings in $\mathcal{S}$.

Definition 1.2. p_{k+m}-#SET PACKING: Given a triple $(\mathcal{S}, k, m)$, where $m = \max\{|s||s \in \mathcal{S}\}$ and $k+m$ is the parameter, count the number of distinct k-packings in $\mathcal{S}$.

Some definitions about parameterized COUNTING MATCHING are described similarly.

Let $T_1, T_2, \ldots, T_m$ be m pairwise disjoint symbol sets. An m-tuple $\rho = (t_1, t_2, \ldots, t_m)$ is called an ordered tuple in $T_1 \times T_2 \times \ldots \times T_m$ if $t_i \in T_i$ $(1 \le i \le m)$. Let $\mathcal{S}$ be a collection of n m-tuples and $M \subseteq \mathcal{S}$. M is a *matching* if any two tuples in M do not intersect. A matching is a k-*matching* if it contains exactly k tuples.

Definition 1.3. p_k-#m-D MATCHING: Given a pair $(\mathcal{S}, k)$, where k is the parameter, count the number of distinct k-matchings in $\mathcal{S}$.

Definition 1.4. p_{k+m}-#D MATCHING: Given a triple $(\mathcal{S}, k, m)$, where $k+m$ is the parameter, count the number of distinct k-matchings in $\mathcal{S}$.

The parameterized COUNTING SUBGRAPH PACKING can be defined similarly. Let $G = (V, E)$ be a simple undirected graph and H a connected graph. Graph G is said to have a *subgraph packing* P_H based on H if there exist k disjoint copies $H_1, H_2, \ldots, H_k$ of H in the vertex set of G. Specially, there are two types of subgraph packing.

Let X be a connected subgraph having m vertices $(m \ge 2)$ and P_X a subgraph packing based on X in G. The subgraph packing P_X is called *vertex-disjoint* if $X_1, X_2, \ldots, X_k$ share no vertices. Clearly, a vertex-disjoint subgraph packing can be considered as a special subgraph having mk vertices.

Definition 1.5. p_k-#m VERTEX-DISJOINT SUBGRAPH PACKING: Given a triple (G, X, k), where k is the parameter, count the number of vertex-disjoint subgraph packings of G isomorphic to P_X.

Let Y be a connected subgraph having m edges $(m \ge 2)$ and P_Y a subgraph packing based on Y in G. The subgraph packing P_Y is called *edge-disjoint* if we allow $Y_1, Y_2, \ldots, Y_k$ to have some vertices in common but no edges exist in $Y_i \cap Y_j$ when $i \neq j$. Note that, for a fixed subgraph Y, there exists a class of edge-disjoint subgraph packings with different shapes according to this relaxed restriction. Without loss of generality, let $\mathcal{C} = \{P_Y^1, P_Y^2, P_Y^3, \ldots, \}$ be the collection of edge-disjoint subgraph packings based on Y.

Definition 1.6. p_k-#m EDGE-DISJOINT SUBGRAPH PACKING: Given a triple (G, Y, k), where k is the parameter, count the number of edge-disjoint subgraph packings of G isomorphic to any subgraph packing in $\mathcal{C}$.

We can similarly define p_{k+m}-#VERTEX/EDGE-DISJOINT SUBGRAPH PACKING. Since these definitions are exactly the same as the definition of p_{k+m}-#SET PACKING, we omit the descriptions for them.

Matching and Packing form a basic class of NP-hard problems. They also have a broad range of applications in many real-world situations [30–32]. Their decision versions are fixed-parameter tractable and were intensively studied in the field of parameterized computation [4,6,7,26–28]. Moreover, for the decision versions of parameterized subgraph packing problems, a series of fixed-parameter tractable algorithms have been developed in recent years [11–13,17,21,25].

On the counting version of parameterized matching and packing, there were also some researches in the past decade. For counting k-packings in a given family of m-sets based on a universal set U, a series of exact algorithms with running time $O^*(|U|^{\lceil mk/2 \rceil})$ were presented in [3,19,29], respectively. An FPTRAS for parameterized counting 3-D MATCHING was developed by Liu et al. [20]. Recently, the complexity of p_{k+m}-#D MATCHING was also discussed by some researchers.

Counting graph matchings, related to some of problems above, has been well-studied in recently years. Specifically, counting k-matchings on general graphs (denoted by COUNTING GRAPH MATCHING) was shown to be #W[1]-hard by Curticapean [8] in 2013. Afterwards, counting k-matchings on bipartite graphs (denoted by COUNTING BIPARTITE GRAPH MATCHING) was also proved to be #W[1]-hard by Curticapean and Marx [9] in 2014. Moreover, an FPTRAS for counting graph matchings was presented by Arvind and Raman [2].