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On Fixed Points of Rational Transductions

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Abstract

We show that it is undecidable whether or not an injective rational function (realized by a finite transducer) $f : A^* \rightarrow A^*$ has a fixed point. The proof applies undecidability of the Post's Correspondence Problem for injective morphisms. As a corollary we obtain that the existence of a fixed point of injective computable functions is undecidable.

1 Introduction

We study the *fixed point problem* of functions starting from the finitely generated word semigroups:

Problem 1. *Let A be a finite alphabet. Let $f : A^* \rightarrow A^*$ be a function. Does there exist a word w such that $f(w) = w$.*

We shall prove that the fixed point problem is undecidable for functions defined by finite transducers, that is, functions defined by finite automata with output. Our result also gives a corollary for computable (recursive) functions over natural numbers.

It easily follows from the basic computability results on Turing machines that the existence of a fixed point is undecidable for general functions. Indeed, this follows from a modification of the classical diagonal argument or by the halting problem, i.e., by defining a function f in a way that it checks for any input i whether the i th Turing machine halts on its code, one can transform f into a function h having fixed point i if and only if the i th Turing machine halts on i . However, in such a construction the function h is clearly noncomputable.

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