



A type of perfect matchings extend to hamiltonian cycles in k -ary n -cubes [☆]

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ABSTRACT

Kreweras conjectured that every perfect matching in a hypercube Q_n for $n \geq 2$ extends to a hamiltonian cycle of Q_n . Fink confirmed the conjecture to be true. The k -ary n -cube Q_n^k is a generalization of the hypercube. However, the analogous result does not necessarily hold for Q_n^k . We can find a perfect matching in Q_2^6 which is not contained in any hamiltonian cycle of Q_2^6 . In this paper, we investigate the existence of a hamiltonian cycle passing through a perfect matching in Q_n^k . For an integer $n \geq 2$ and an even integer $k \geq 6$, we prove that every perfect matching in Q_n^k consisting of edges in the same dimension can be extended to a hamiltonian cycle of Q_n^k .

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1. Introduction

The interconnection network is usually represented by a graph where vertices represent processors and edges represent communication links between processors. The k -ary n -cube is one of the most popular interconnection networks for parallel and distributed computer systems because of its desirable properties, such as ease of implementation, low-latency and high-bandwidth interprocessor communication [2,6,15].

The k -ary n -cube, denoted by Q_n^k ($n \geq 1$ and $k \geq 2$), is a graph consisting of k^n vertices, each of which has the form $u = \alpha_1 \cdots \alpha_n$, where $0 \leq \alpha_i \leq k-1$ for every $i \in \{1, \dots, n\}$. Two vertices $u = \alpha_1 \cdots \alpha_n$ and $v = \beta_1 \cdots \beta_n$ are adjacent if and only if there exists an integer $j \in \{1, \dots, n\}$ such that $\alpha_j = \beta_j \pm 1 \pmod{k}$ and $\alpha_i = \beta_i$ for every $i \in \{1, \dots, n\} \setminus \{j\}$. Such an edge uv is called a j -dimensional edge. For clarity of presentation, we omit writing “(mod k)” in similar expressions for the remainder of the paper. The Q_n^2 is the well-studied hypercube, denoted by Q_n . The Q_n^4 is isomorphic to the graph Q_{2n} . The Q_1^k with $k \geq 3$ is a cycle of length k . It is well known that Q_n^k is hamiltonian except the case that $n = 1$ and $k = 2$.

Since some parallel applications such as those in image and signal processing are originally designated for a path or cycle architecture, it is important to investigate cycle and path embeddings in an interconnection network [1,4,5,7,8,11,12,17,19,20].

Ruskey and Savage [14] asked the following question: Does every matching in Q_n for $n \geq 2$ extend to a hamiltonian cycle of Q_n ? Kreweras [13] conjectured that every perfect matching of Q_n for $n \geq 2$ can be extended to a hamiltonian cycle of Q_n . Fink [9,10] confirmed the conjecture to be true.

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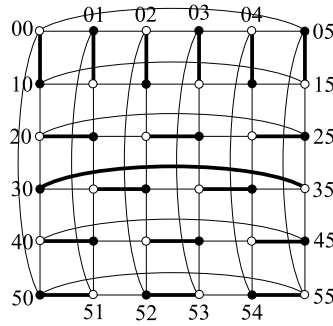


Fig. 1. Perfect matching M which is not contained in any hamiltonian cycle of Q_2^6 .

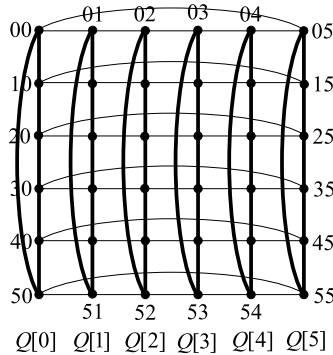


Fig. 2. Split Q_2^6 into $Q[0], \dots, Q[5]$ at position 2.

However, the analogous result does not necessarily hold for Q_n^k . We can find a perfect matching M in Q_2^6 as shown in Fig. 1 which is not contained in any hamiltonian cycle of Q_2^6 . We can verify this by a computer program. See Appendix A. Note that M may not be the only perfect matching not contained in any hamiltonian cycle of Q_2^6 . When n is larger and k is smaller, we guess there are many such perfect matchings because the degree of the vertices is smaller.

On the other hand, Caha and Koubek [3] investigated the hamiltonian cycle embedding problem in the hypercube with prescribed edges. Dvořák [7] showed for $n \geq 2$ that every linear forest containing at most $2n - 3$ edges of a hypercube can be embedded into a hamiltonian cycle of Q_n , where a forest is *linear* if each component of it is a path. Wang et al. [18] and Stewart [16] proved for $n \geq 2$ and $k \geq 3$ that every linear forest containing at most $2n - 1$ edges in Q_n^k can be embedded into a hamiltonian cycle in Q_n^k .

Given a perfect matching in a k -ary n -cube, which conditions guarantee the existence of a hamiltonian cycle passing through the perfect matching? For an integer $n \geq 2$ and an even integer $k \geq 6$, we prove that every perfect matching in Q_n^k consisting of edges in the same dimension can be extended to a hamiltonian cycle of Q_n^k .

2. Definitions and preliminaries

The vertex set and edge set of a graph G are denoted by $V(G)$ and $E(G)$, respectively. For a set $F \subseteq E(G)$, let $G - F$ denote the resulting graph after removing all edges in F from G . Let H and H' be two subgraphs of G . We use $H + H'$ to denote the graph with the vertex set $V(H) \cup V(H')$ and edge set $E(H) \cup E(H')$. For $F \subseteq E(G)$, we use $H + F$ to denote the graph with the vertex set $V(H) \cup V(F)$ and edge set $E(H) \cup F$, where $V(F)$ denotes the set of vertices incident with F .

A set of edges in a graph G is called a *matching* if no two edges have an endpoint in common. A matching in G is *perfect* if it covers all vertices of G . Obviously, a Q_n^k with odd k has no perfect matching. A cycle in a graph G is a *hamiltonian cycle* if every vertex in G appears exactly once in the cycle. Let M be a matching in G . An M -*alternating path* (resp. *cycle*) in G is a path (resp. cycle) whose edges are alternately in $E(G) \setminus M$ and M .

For $d \in \{1, \dots, n\}$, let E_d denote the set of all d -dimensional edges in Q_n^k . Then $E(Q_n^k) = \bigcup_{d=1}^n E_d$. Let $Q_{n-1}^{k,d}[j]$, $j \in \{0, 1, \dots, k-1\}$, be the $(n-1)$ -dimensional subcube of Q_n^k induced by all the vertices with the d th position being j , abbreviated as $Q[j]$ if there are no ambiguities. Clearly, $Q_n^k - E_d = Q[0] + Q[1] + \dots + Q[k-1]$. We say that Q_n^k *splits* into $(n-1)$ -dimensional subcubes $Q[0], Q[1], \dots, Q[k-1]$ at position d . See Fig. 2 for example.

The *parity* $p(u)$ of a vertex $u = \alpha_1 \dots \alpha_n$ in Q_n^k with even k is defined by $p(u) = \sum_{i=1}^n \alpha_i \pmod{2}$. Then there are $\frac{k^n}{2}$ vertices with parity 0 and $\frac{k^n}{2}$ vertices with parity 1 in Q_n^k . Observe that Q_n^k with even k is bipartite and vertices of each parity form bipartite sets of Q_n^k .

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