Doctopic: Algorithms, automata, complexity and games ARTICLE IN PRESS TCS:11512

Theoretical Computer Science ••• (••••) •••–•••

Contents lists available at ScienceDirect

Theoretical Computer Science

www.elsevier.com/locate/tcs

Complexity of manipulation with partial information in voting

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ARTICLE INFO

Article history:
Received 12 July 2017
Received in revised form 13 December 2017
Accepted 7 March 2018
Available online xxxx
Communicated by X. Deng

Keywords:
Voting
Manipulation
Incomplete information
Algorithm
Computational complexity

ABSTRACT

The Coalitional Manipulation problem has been studied extensively in the literature for many voting rules. However, most studies have focused on the complete information setting, wherein the manipulators know the votes of the non-manipulators. While this assumption is reasonable for purposes of showing intractability, it is unrealistic for algorithmic considerations. In most real-world scenarios, it is impractical to assume that the manipulators to have accurate knowledge of all the other votes. In this work, we investigate manipulation with incomplete information. In our framework, the manipulators know a partial order for each voter that is consistent with the true preference of that voter. In this setting, we formulate three natural computational notions of manipulation, namely weak, opportunistic, and strong manipulation. We say that an extension of a partial order is *viable* if there exists a manipulative vote for that extension. We propose the following notions of manipulation when manipulators have incomplete information about the votes of other voters.

1. Weak Manipulation: the manipulators seek to vote in a way that makes their preferred candidate win in *at least one extension* of the partial votes of the non-manipulators.
2. Opportunistic Manipulation: the manipulators seek to vote in a way that makes their preferred candidate win *in every viable extension* of the partial votes of the non-manipulators.
3. Strong Manipulation: the manipulators seek to vote in a way that makes their preferred candidate win *in every extension* of the partial votes of the non-manipulators.

We consider several scenarios for which the traditional manipulation problems are easy (for instance, Borda with a single manipulator). For many of them, the corresponding manipulative questions that we propose turn out to be computationally intractable. Our hardness results often hold even when very little information is missing, or in other words, even when the instances are very close to the complete information setting. Our results show that the impact of paucity of information on the computational complexity of manipulation crucially depends on the notion of manipulation under consideration. Our overall conclusion is that computational hardness continues to be a valid obstruction to manipulation, in the context of a more realistic model.

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https://doi.org/10.1016/j.tcs.2018.03.012
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Please cite this article in press as: P. Dey et al., Complexity of manipulation with partial information in voting, Theoret. Comput. Sci. (2018), https://doi.org/10.1016/j.tcs.2018.03.012

1. Introduction

In many real life and AI related applications, agents often need to agree upon a common decision although they have different preferences over the available alternatives. A natural tool used in these situations is voting. Some classic examples of the use of voting rules in the context of multiagent systems include Clarke tax [27], collaborative filtering [43], and similarity search [33], etc. In a typical voting scenario, we have a set of candidates and a set of voters reporting their rankings of the candidates called their preferences or votes. A voting rule selects one candidate as the winner once all voters provide their votes. A set of votes over a set of candidates along with a voting rule is called an election. A central issue in voting is the possibility of *manipulation*. For many voting rules, it turns out that even a single vote, if cast differently, can alter the outcome. In particular, a voter manipulates an election if, by misrepresenting her preference, she obtains an outcome that she prefers over the "honest" outcome. In a cornerstone impossibility result, Gibbard and Satterthwaite [35,46] show that every unanimous and non-dictatorial voting rule with three candidates or more is manipulable. We refer to [2] for an excellent introduction to various strategic issues in computational social choice theory.

Considering that voting rules are indeed susceptible to manipulation, it is natural to seek ways by which elections can be protected from manipulations. The works of Bartholdi et al. [6,5] approach the problem from the perspective of computational intractability. They exploit the possibility that voting rules, despite being vulnerable to manipulation in theory, may be hard to manipulate in practice. Indeed, a manipulator is faced with the following decision problem: given a collection of votes $\mathcal{P}$ and a distinguished candidate c, does there exist a vote v that, when tallied with $\mathcal{P}$, makes c win for a (fixed) voting rule r? The manipulation problem has subsequently been generalized to the problem of Coalitional manipulation by Conitzer et al. [12], where one or more manipulators collude together and try to make a distinguished candidate win the election. The manipulation problem, fortunately, turns out to be NP-hard in several settings. This established the success of the approach of demonstrating a computational barrier to manipulation.

However, despite having set out to demonstrate the hardness of manipulation, the initial results in [6] were to the contrary, indicating that many voting rules are in fact easy to manipulate. Moreover, even with multiple manipulators involved, popular voting rules like plurality, veto, k-approval, Bucklin, and Fallback continue to be easy to manipulate [52]. While we know that the computational intractability may not provide a strong barrier [44,45,32,49,50,28,47,48,38,14,22,23,21,24,25,17,18,16] even for rules for which the coalitional manipulation problem turns out to be NP-hard, in all other cases the possibility of manipulation is a much more serious concern.

1.1. Motivation and problem formulation

In our work, we propose to extend the argument of computational intractability to address the cases where the approach appears to fail. We note that most incarnations of the manipulation problem studied so far are in the complete information setting, where the manipulators have complete knowledge of the preferences of the truthful voters. While these assumptions are indeed the best possible for the computationally negative results, we note that they are not reflective of typical real-world scenarios. Indeed, concerns regarding privacy of information, and in other cases, the sheer volume of information, would be significant hurdles for manipulators to obtain complete information. Motivated by this, we consider the manipulation problem in a natural *partial information* setting. In particular, we model the partial information of the manipulators about the votes of the non-manipulators as partial orders over the set of candidates. A partial order over the set of candidates will be called a partial vote. Our results show that several of the voting rules that are easy to manipulate in the complete information setting become intractable when the manipulators know only partial votes. Indeed, for many voting rules, we show that even if the ordering of a small number of pairs of candidates is missing from the profile, manipulation becomes an intractable problem. Our results therefore strengthen the view that manipulation may not be practical if we limit the information the manipulators have at their disposal about the votes of other voters [13].

We introduce three new computational problems that, in a natural way, extend the question of manipulation to the partial information setting. In these problems, the input is a set of partial votes $\mathcal{P}$ corresponding to the votes of the non-manipulators, a non-empty set of manipulators M, and a preferred candidate c. The task in the Weak Manipulation (WM) problem is to determine if there is a way to cast the manipulators' votes such that c wins the election for at least one extension of the partial votes in $\mathcal{P}$. On the other hand, in the Strong Manipulation (SM) problem, we would like to know if there is a way of casting the manipulators' votes such that c wins the election in *every extension* of the partial votes in $\mathcal{P}$.

We also introduce the problem of Opportunistic Manipulation (OM), which is an "intermediate" notion of manipulation. Let us call an extension of a partial profile *viable* if it is possible for the manipulators to vote in such a way that the manipulators' desired candidate wins in that extension. In other words, a viable extension is a Yes-instance of the standard Coalitional Manipulation problem. We have an opportunistic manipulation when it is possible for the manipulators to cast a vote which makes c win the election in *all* viable extensions. Note that any Yes-instance of Strong Manipulation is also an Yes-instance of Opportunistic Manipulation, but this may not be true in the reverse direction. As a particularly extreme example, consider a partial profile where there are no viable extensions: this would be a No-instance for Strong Manipulation, but a (vacuous) Yes-instance of Opportunistic Manipulation. The Opportunistic Manipulation problem allows us to explore a more relaxed notion of manipulation: one where the manipulators are obliged to be successful only in extensions where it is possible to be successful. Note that the goal with Strong Manipulation is to be successful in all extensions, and therefore the only interesting instances are the ones where all extensions are viable.

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