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Team assembling problem for asynchronous heterogeneous mobile robots ☆

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A B S T R A C T

We investigate the *team assembling problem* for a swarm of heterogeneous mobile robots which requires the robots to autonomously partition themselves into teams satisfying a given specification $A = (a_1, a_2, \dots, a_k)$, where a_i is the number of robots with color (i.e., robot type) i in one team. A robot, which is represented by a point in the two-dimensional Euclidean space, is asynchronous, oblivious, and anonymous in the sense that robots with the same color are indistinguishable and all robots execute the same algorithm to determine their moves. It has neither any access to the global coordinate system nor any explicit communication medium. We show that $GCD(a_1, a_2, \dots, a_k) = 1$ is a necessary and sufficient condition for the robots to have an algorithm to solve the team assembling problem in a *self-stabilizing* manner, i.e., starting from any arbitrary initial configuration, the robots form teams according to the specification.

1. Introduction

As coordinated behavior of a team consisting of *heterogeneous* mobile robots with different capabilities has gained much attention, the importance of automated self-assembly of such a team satisfying a specification is gradually getting recognized. However, the state of the art of this research direction still goes no further than forming teams consisting of *homogeneous* robots [15,29], although many projects have been conducted to produce robot teams [3,13,14,25,26,34]. We formally investigate this problem of autonomous *team assembling* by heterogeneous mobile robots to organize teams each satisfying a given specification as a motion planning problem in distributed computing.

In distributed computing, Sugihara and Suzuki [31] first investigated the motion planning problem for a swarm of homogeneous robots, and later proposed an algorithm for partitioning n robots into m groups with the same size n/m [32]. However their partition algorithm is heuristic; they did not discuss the correctness of the algorithm since a formal model of mobile robot system was not defined in [31,32], and the algorithm requires assistance of an outside supervisor to move a few robots. A formal model of a homogeneous mobile robot system was proposed for semi-synchronous (SSYNC) and fully synchronous (FSYNC) robots in [33] and for asynchronous (ASYNC) robots in [21], which are now considered to be standard models of mobile robot systems in distributed computing (see e.g., [19]). Under these models, provided $n > m > 1$, the

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partition problem is unsolvable for arbitrary values of n and m [18,23,35], which partly explains why a partition algorithm (not a heuristic one) was not proposed in [32]. Note that the problem becomes trivial when $n = m$ or $m = 1$.

In this paper, we investigate the *team assembling problem* for a swarm of *heterogeneous* mobile robots, under the ASYNC robot model. We extend the original ASYNC robot model to a heterogeneous ASYNC robot model by introducing for each robot a color to specify its type. A robot is represented by a point in the two-dimensional Euclidean space and has a color selected from a domain $\{1, 2, \ldots, k\}$. It repeats executing a "Look-Compute-Move" cycle, during which it observes the positions and the colors of all robots (Look), computes its next position and the route to it using a common deterministic algorithm (Compute), and traces the route toward the next position (Move). The travel to the next position may be truncated en route by an adversary. We do not make any assumptions on the timing and the length of a Look-Compute-Move cycle except that the length of each Look-Compute-Move cycle is finite.

Each robot has no access to the global coordinate system and it uses a *local x-y coordinate system*, whose origin always indicates its position, but the unit distance and the direction of the positive x-axis never change. Then the actions taken by a robot are in terms of its x-y local coordinate system. The robots have *chirality*; they have a sense of clockwise direction. We thus assume without loss of generality that they have a sense of *right-handedness*, and the global and all local x-y coordinate systems are right-handed.

Each robot is *anonymous* and two robots are distinguished just by their positions and colors. We assume that it has the *strong multiplicity detection ability* and can count, for each color, the number of robots with the color at a point. Each robot is *oblivious* in the sense that it has no memory to memorize its execution history and the input to an algorithm is the positions and the colors of all robots that it has observed in the immediately preceding Look. Thus a *configuration* of the robots consists of the set of their positions and their colors in terms of the global coordinate system, i.e., a multiset of colored points. Finally, the robots have no communication devices to exchange information.

Now we define the *team assembling problem* for robots with k colors. Given a specification $A = (a_1, a_2, \ldots, a_k)$, we want to design an algorithm for the robots to form teams, each of which contains exactly a_i robots with color i for each $i = 1, 2, \ldots, k$, by each team gathering at a distinct point starting from *any* initial configuration. By the definition of the team assembling problem, any team assembling algorithm is a *self-stabilizing* algorithm [12] that tolerates any finite number of transient faults by making the system regain its correct behavior from an arbitrary initial configuration. The trick in a self-stabilizing algorithm is that the configuration after the final transient fault is considered as an arbitrary initial configuration.[1]

We first assume that there exists an integer $m > 1$ such that the number of robots with color i is $a_i m$ for each color $i = 1, 2, \ldots, k$. When $m = 1$, the problem is reduced to the gathering problem for oblivious ASYNC robots, which is solvable when $n > 2$ [8]. Here we remark that the team assembling problem for $k = 1$ coincides with the partition problem for homogeneous robots, which is unsolvable even for non-oblivious FSYNC robots as mentioned above. We later discuss a variant of the team assembling problem in which the number of robots for each color is not exact and the robots are required to form as many teams as possible.

1.1. Our contribution

The goal of this paper is to show the next theorem.

Theorem 1. *Given a specification $A = (a_1, a_2, \ldots, a_k)$ for the team assembling problem, consider a set of oblivious heterogeneous ASYNC robots consisting of $a_i m$ color i robots for each $i = 1, 2, \ldots, k$ for some integer $m > 1$. Then the robots can form teams with respect to A from any initial configuration where no two robots with the same color occupy the same position if and only if $GCD(a_1, a_2, \ldots, a_k) = 1$.*

For example, the team assembling problem is unsolvable for instances $A_1 = (6, 10)$, $A_2 = (10, 15)$, and $A_3 = (15, 6)$, but is solvable for an instance $A_4 = (6, 10, 15)$ since $GCD(6, 10, 15) = 1$. To show the sufficiency of Theorem 1, we present a team assembling algorithm for oblivious ASYNC robots. The proposed algorithm can form e.g., 100 teams each satisfying A_4 from *any* initial configuration, if there are 600, 1000 and 1500 robots with colors 1, 2 and 3, respectively.

When $k = 1$, Theorem 1 means that the robots cannot form multiple teams consisting of more than one member, because $GCD(a_1) = a_1$. Thus Theorem 1 is a generalization of the partitioning problem for homogeneous robots [18].

1.2. Related works

As models of homogeneous mobile robots, fully synchronous (FSYNC) and semi-synchronous (SSYNC) robots were introduced in [33], and asynchronous (ASYNC) robots in [21]. For all $t \geq 1$, FSYNC robots start the t-th Look-Compute-Move cycle simultaneously and execute each of its Look, Compute and Move synchronously. SSYNC robots behave exactly the same as FSYNC robots, except that some of them may not start the i-th Look-Compute-Move cycle for some i. No assumptions are

[1] Generally, a transient fault corrupts states of computing entities without changing their IDs (if any) and their local algorithm codes. We note that in our model, the robots are state-less (oblivious) and the color of a robot is not its state.

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