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Realizability of graphs as triangle cover contact graphs

Shaheena Sultana [*,1], Md. Saidur Rahman

Graph Drawing and Information Visualization Laboratory, Department of Computer Science and Engineering, Bangladesh University of Engineering and Technology (BUET), Dhaka-1000, Bangladesh

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ABSTRACT

Let $S = \{p_1, p_2, \ldots, p_n\}$ be a set of pairwise disjoint geometric objects of some type and let $C = \{c_1, c_2, \ldots, c_n\}$ be a set of closed objects of some type with the property that each element in C covers exactly one element in S and any two elements in C can intersect only on their boundaries. We call an element in S a *seed* and an element in C a *cover*. A *cover contact graph (CCG)* consists of a set of vertices and a set of edges where each of the vertex corresponds to each of the covers and each edge corresponds to a connection between two covers if and only if they touch at their boundaries. A *triangle cover contact graph (TCCG)* is a cover contact graph whose cover elements are triangles. In this paper, we show that every Halin graph has a realization as a $TCCG$ on a given set of collinear seeds. We introduce a new class of graphs which we call super-Halin graphs. We also show that the classes super-Halin graphs, cubic planar Hamiltonian graphs and $a \times b$ grid graphs have realizations as $TCCG$s on collinear seeds. We also show that some non-planar graphs have planar realizations as $TCCG$s. Every complete graph and clique-3 graph have realizations as $TCCG$s on any given set of seeds. Note that only trees and cycles are known to be realizable as CCGs and outerplanar graphs are known to be realizable as $TCCG$s.

1. Introduction

Let $S = \{p_1, p_2, \ldots, p_n\}$ be a set of pairwise disjoint geometric objects of some type and $C = \{c_1, c_2, \ldots, c_n\}$ be a set of closed objects of some type with the property that each element in C covers exactly one element in S and any two elements in C can intersect only on their boundaries. We call an element in S a *seed* and an element in C a *cover*. The seeds may be points, disks or triangles and covering elements may be disks or triangles. The *cover contact graph (CCG)* consists of a set of vertices and a set of edges where each of the vertex corresponds to each of the covers and each edge corresponds to a connection between two covers if they touch at their boundaries. In other words, two vertices of a cover contact graph are adjacent if and only if the corresponding cover elements touch at their boundaries. Note that the vertices of the cover contact graph are in one-to-one correspondence to both seeds and covering objects. In a cover contact graph, if disks are used as covers then it is called a *disk cover contact graph* and if triangles are used as covers then it is called a *triangle cover contact graph (TCCG)*. Fig. 1(b) depicts the disk cover contact graph induced by the disk covers in Fig. 1(a), whereas Fig. 1(d) depicts the triangle cover contact graph induced by the triangle covers in Fig. 1(c).

* Corresponding author.
E-mail addresses: zareefas.sultana@gmail.com (S. Sultana), saidurrahman@cse.buet.ac.bd (Md. Saidur Rahman).

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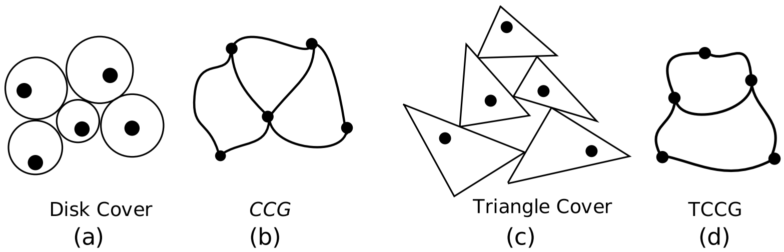

Fig. 1. Illustration for CCG and $TCCG$.

There are several works [1–5] in the area of geometric optimization where the problem is how to cover geometric objects such as points by other geometric objects such as disks. The main goal is to minimize the radius of a set of k disks to cover n input points. Applications of such covering problems are found in geometric optimization problems such as facility location problems [1,2]. There are also lot of works [2,6] related to optimization of covering geometric objects. Abellanas et al. [7] illustrated *coin placement problem*, which is NP-complete. They tried to cover the n points using n disks (each having different radius) by placing each disk in the center position at one of the points so that no two disks overlap. Further Abellanas et al. [8] considered another related problem. They showed that for a given set of points in the plane, it is also NP-complete to decide whether there are disjoint disks centered at the points such that the contact graph of the disks is connected.

Atienza et al. [9] introduced the concept of cover contact graphs where they considered a problem which they called "realization problem." They gave some necessary conditions and then showed that it is NP-hard to decide whether a given graph can be realized as a disk cover contact graph if the correspondence between vertices and point seeds is given. They also showed that every tree and cycle have realizations as CCGs on a given set of collinear point seeds. Recently, Iqbal et al. [10] worked on triangle cover contact graphs ($TCCG$s) where the seeds are points and the covers are triangles. First they considered the set of seeds which are in general position, i.e., no two seeds lie on a vertical line and they gave an $O(n \log n)$ algorithm to construct a 3-connected $TCCG$ of the set of seeds. They also gave an $O(n \log n)$ algorithm to construct a 4-connected $TCCG$ for a given set of six or more seeds. Addressing the realization problem, they gave an algorithm that realizes a given outerplanar graph as a $TCCG$ for a given set of seeds on a line.

In this paper, addressing the realization problem we show that every Halin graph has a realization as a $TCCG$ on a given set of collinear seeds. We introduce a new class of graphs which we call super-Halin graphs. We also show that the classes super-Halin graphs, cubic planar Hamiltonian graphs and $a \times b$ grid graphs have realizations as $TCCG$s on collinear seeds. We also show that every complete graph and clique-3 graph which are non-planar graphs have realizations as $TCCG$s on any given set of seeds. The concept of the realization of non-planar graphs as triangle cover contact graphs is interesting because realization as $TCCG$s of non-planar graphs are planar. This realization can be used in VLSI layout design. A multilayer circuit can be implemented in a single layer.

The remaining of the paper is organized as follows. Section 2 presents some definitions and preliminary results. Section 3 gives an algorithm that realizes a given Halin graph as a $TCCG$. Section 4 introduces a new class of graphs which we call super-Halin graphs and gives an algorithm that realizes a super-Halin graph as a $TCCG$. Two algorithms that realize a given Hamiltonian graph and an $a \times b$ grid graph as $TCCG$s are given in Section 5 and Section 6, respectively. In Section 7 we present two algorithms that realize two classes of non-planar graphs as planar $TCCG$s. Finally, Section 8 contains concluding remarks and directions for further research in this field. Some preliminary results of this paper were presented at COCOA 2016 [11].

2. Preliminaries

In this section we present some terminologies and definitions which will be used throughout the paper. For the graph theoretic definitions which have not been described here, see [12,13].

A graph is *planar* if it can be embedded in the plane without edge crossing except at the vertices where the edges are incident. A *plane graph* is a planar graph with a fixed planar embedding. A plane graph divides the plane into connected regions called *faces*. The unbounded region is called the *outer face*; the other faces are called *inner faces*. The cycle lies on the outer face is called *outer cycle*. We denote the outer cycle of G by $C_o(G)$. The edges in the outer cycle is called *outer edges*.

A graph G is *connected* if there is a path between any two distinct vertices u and v in G. A graph which is not connected is called a *disconnected graph*. Let $G = (V, E)$ be a connected simple graph with vertex set V and edge set E. A *subgraph* of a graph $G = (V, E)$ is a graph $G' = (V', E')$ such that $V' \subseteq V$ and $E' \subseteq E$. If G' contains all the edges of G that join vertices in V', then G' is called the subgraph *induced* by V'. The *connectivity* $\kappa(G)$ of a graph G is the minimum number of vertices whose removal results in a disconnected graph or a single-vertex graph. We say that G is k-connected if $\kappa(G) \geq k$. A vertex v in G is a *cut-vertex* if the removal of v results in a disconnected graph. A *biconnected component* is a maximal biconnected subgraph.