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On the construction of graphs with a planar bipartite double cover from boolean formulas and its application to counting satisfying solutions

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Abstract

We address the problem to construct a graph $\mathcal{G}$ with binary edge weights from the incidence graph of a boolean formula Φ in a way, that $\mathcal{G}$ has a *planar bipartite double cover* $\ddot{\mathcal{G}}$. This allows one to use the identity between the permanent of $\mathcal{G}$'s adjacency matrix $\mathbf{A}_{\mathcal{G}}$ and the number of perfect matchings of $\ddot{\mathcal{G}}$, i.e., $\mathrm{perm}(\mathbf{A}_{\mathcal{G}}) = \mathrm{pm}(\ddot{\mathcal{G}})$. Due to the algorithm of Fisher, Kasteleyn and Temperlay (FKT), the right hand side of the equation can be counted in polynomial time if $\ddot{\mathcal{G}}$ is planar.

We prove a theorem that describes necessary conditions for the gadgets (i.e., small subgraphs with certain properties) of $\mathcal{G}$ to allow a planar bipartite double cover $\ddot{\mathcal{G}}$. For arbitrary 3CNF formulas and the known approaches to build $\mathcal{G}$, our theorem shows that gadgets, which enable a planar bipartite double cover, do not exists. However, we show that for certain classes of boolean formulas, e.g., $\#_5$Pl-Rtw-2CNF, $\#_7$Pl-Rtw-2CNF and $\#_3$Forest-3CNF, such gadgets exists, hence these counting problems are in P. The results probably extended to other moduli p and perhaps further counting classes.

Keywords: Planar graph, bipartite double cover, perfect matchings, permanent, counting satisfying solutions

1. Introduction

In 1976 Leslie Valiant [1] proved that the complexity to compute the permanent of a matrix is #P-complete. Based on a given 3CNF formula Φ, he constructed a graph $\mathcal{G}$ such that the permanent of $\mathcal{G}$'s adjacency matrix $\mathbf{A}_{\mathcal{G}}$ contains the number of satisfying solutions $\#\Phi$. Later in 1993, Ben-Dor with Halevi [2] simplified his approach by reducing the number of necessary graph gadgets. Additionally, they showed how to reduce the matrix $\mathbf{A}_{\mathcal{G}}$ to a 0/1-matrix and concurrently preserving the permanent value.

However, it is well known, that the permanent of a 0/1-matrix is equal to the number of perfect matchings in a graph's *bipartite double cover* $\ddot{\mathcal{G}}$. As long as a graph is planar, perfect matchings can be counted in polynomial time using the *FKT algorithm* [3, 4]. So it is obvious to ask the following question:

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