Theoretical Computer Science ••• (••••) •••–•••

Contents lists available at ScienceDirect

Theoretical Computer Science

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Parameterized complexity of team formation in social networks ☆

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A R T I C L E I N F O

Article history:
Received 15 November 2016
Received in revised form 15 May 2017
Accepted 21 May 2017
Available online xxxx

Keywords:
Computational complexity analysis
Parameterized complexity analysis
Team formation on social networks

A B S T R A C T

Given a task that requires some skills and a social network of individuals with different skills, the Team Formation problem asks to find a team of individuals that together can perform the task, while minimizing communication costs. Since the problem is NP-hard, we identify the source of intractability by analyzing its parameterized complexity with respect to parameters such as the total number of skills k, the team size l, the communication cost budget b, and the maximum vertex degree Δ. We show that the computational complexity strongly depends on the communication cost measure: when using the weight of a minimum spanning tree of the subgraph formed by the selected team, we obtain fixed-parameter tractability for example with respect to the parameter k. In contrast, when using the diameter as measure, the problem is intractable with respect to any single parameter; however, combining Δ with either b or l yields fixed-parameter tractability.

1. Introduction

Assembling teams based on required skills is a classic management task. Recently, it has been suggested to take into account not only the covering of the required skills, but also the expected communication costs (see Lappas et al. [13] for a survey). This cost can be estimated based on a given edge-weighted social network, where a low weight value on an edge between two individuals indicates a low communication cost. For example, edge weights can reflect distance in an organizational chart or the number of joint projects completed.

Lappas et al. [12] formalized the setting as the optimization problem of minimizing the communication cost and studied two cost measures: the diameter (Diam) and the weight of a minimum spanning tree (Mst). For our complexity analysis, we formulate it as a decision problem by fixing the maximum team size.

Diam-Team Formation
Input: An undirected graph $G = (V, E)$ with edge-weight function $w\colon E \to \mathbb{N}$, a set T of k skills, a skill function $S\colon V \to 2^T$, a team size $l \in \mathbb{N}$, and a budget $b \in \mathbb{N}$.
Question: Is there a subset $V' \subseteq V$ with $|V'| \le l$ such that $\bigcup_{v \in V'} S(v) = T$ and the w-weighted diameter of the induced subgraph $G[V']$ is at most b?

☆ An extended abstract version appeared in the proceedings of the 11th International Conference on Algorithmic Aspects in Information and Management (AAIM), Bergamo, Italy, July of 2016.

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http://dx.doi.org/10.1016/j.tcs.2017.05.024
0304-3975/

Please cite this article in press as: R. Bredereck et al., Parameterized complexity of team formation in social networks, Theoret. Comput. Sci. (2017), http://dx.doi.org/10.1016/j.tcs.2017.05.024

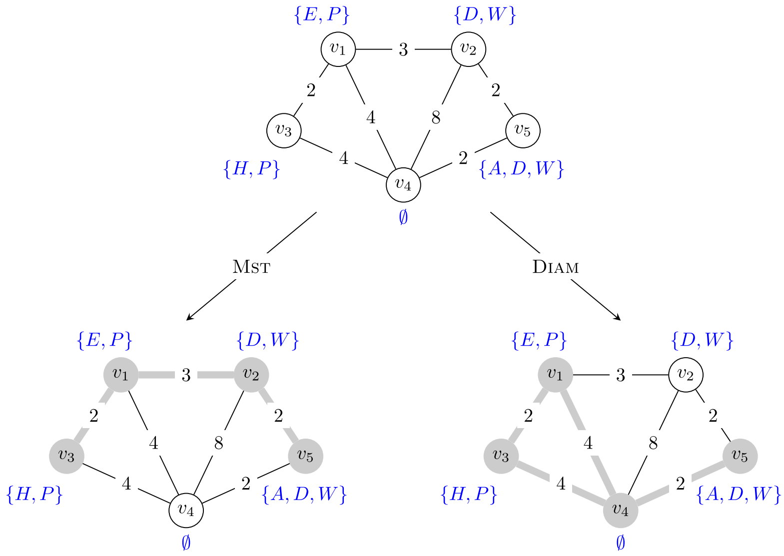

Fig. 1. A Team Formation example: The upper figure shows a social network of five potential team members and six skills, "algorithms" (A), "data bases" (D), "software engineering" (E), "hardware support" (H), "programming" (P), and "web programming" (W). When minimizing the weight of a minimum spanning tree of the subgraph induced by a team (Mst), the team with members v_1, v_2, v_3, v_5 has the lowest cost, 7 (see the lower left figure). However, when minimizing the diameter of the subgraph induced by a team (Diam), it is worthwhile to replace v_2 with v_4—who has no specific skill—to reduce the diameter of 7 in $G[\{v_1, v_2, v_3 v_5\}]$ to the diameter of 6 in $G[\{v_1, v_3, v_4, v_5\}]$ (see the lower right figure).

Here, the *diameter* of an edge-weighted graph G, denoted as Diam(G), is the maximum distance between any two vertices in the input graph and the *distance* between two vertices is the minimum sum of the weights of the edges along any path between these two vertices. Our formulation of the team formation problem allows to choose individuals (vertices) that do not contribute any skills, but serve as intermediate vertices to lower overall communication costs. We further assume w.l.o.g. that no individual has a skill that is not in the request set T.

The *weight of a minimum spanning tree* of graph G, Mst(G), is the smallest sum of the weights of the edges in any spanning tree of G. We define the corresponding Mst-Team Formationproblem by replacing "diameter" in the definition of Diam-Team Formationwith "weight of a minimum spanning tree".

Fig. 1 illustrates an example for the Diam-Team Formation and Mst-Team Formation problems. Lappas et al. [12] showed that both problems are NP-complete. Experiments on Diam-Team Formation, Mst-Team Formation and similar team formation problems so far use heuristic algorithms [1,5,12,9,14]. However, it might be that instances encountered in practice are actually easier than a one-dimensional complexity analysis suggests, and can be solved optimally. For example, it might be reasonable to assume that only a small number of skills is required. Thus, we try to identify the sources of intractability through a parameterized complexity analysis.

1.1. Optimization variant

There are two natural ways to define approximate solutions of our team formation problem. First, to allow solutions with larger communication costs. This leads to the MinCost-ζ-Team Formation problem, ζ being either Diam or Mst, which asks for a vertex subset $V' \subseteq V$ with $|V'| \le l$ such that $\bigcup_{v \in V'} S(v) = T$ and the communication cost $\zeta(G[V'])$ is minimized. Second, to allow solutions with larger teams. This leads to the MinTeamsize-ζ-Team Formation problem, which asks for a minimum vertex subset $V' \subseteq V$ such that $\bigcup_{v \in V'} S(v) = T$ and $\zeta(G[V']) \le b$.

Cost measure "diameter" Arkin and Hassin [2] studied MinCost-Diam-Team Formation with unlimited team size l under the name Multiple-Choice Cover. They showed that even when no skill is allowed to be covered by more than three team members, the problem still cannot be approximated with a constant-factor error guarantee, unless P = NP. However, when the weights satisfy the triangle inequality, a 2-approximation is possible; this bound is sharp [2].

Cost measure "minimum spanning tree" As already mentioned by Lappas et al. [12], the MinCost-Mst-Team Formation problem with an unlimited team size l is equivalent to the Group Steiner Tree problem: given an undirected edge-weighted graph $G = (V, E)$ and vertex subsets (*groups*) $g_i \subseteq V$, $1 \le i \le k$, find a subtree $T = (V_T, E_T)$ of G such that $V_T \cap g_i \neq \emptyset$ for all $1 \le i \le k$ and the cost $\sum_{e \in E_T} w(e)$ is minimized. Clearly, each group of Group Steiner Tree corresponds to a subset of vertices in Mst-Team Formation that have a particular skill. From this relation to Group Steiner Tree and an inapproximability result by Halperin and Krauthgamer [11], we obtain that it is unlikely that MinCost-Mst-Team Formation can be approximated to a factor of $O(\log^{2-\epsilon} k)$ for any $\epsilon > 0$, where k is the number of skills to be covered.

Please cite this article in press as: R. Bredereck et al., Parameterized complexity of team formation in social networks, Theoret. Comput. Sci. (2017), http://dx.doi.org/10.1016/j.tcs.2017.05.024