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On network formation games with heterogeneous players and basic network creation games ☆

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We consider two variants of the network formation game that aims to study the creation of large-scale networks and to capture the impact of selfish behavior, on behalf of the network administrators, on the overall network structure and performance. In particular, we study basic network creation games, where each player aims to minimize her distance to the remaining players, and we present an improved lower bound on the graph diameter of equilibria of this game. We also consider network formation games with a large number of heterogeneous players and monetary transfers, and prove tight bounds on the price of anarchy under realistic assumptions about the cost function. Finally, we argue about the setting where these heterogeneous players must be connected with additional edge-disjoint paths to mitigate the impact of failures.

1. Introduction

The advent and widespread adoption of the Internet has given rise to questions like how such large-scale Internet-like networks are formed and sustained. Since such a network mainly relies on the voluntary participation of its members, the need to understand the motives behind network administrators' actions arises quite naturally. Being part of a large-scale network offers great benefits to the participating entities, who, however, would prefer to enjoy these benefits with the minimum possible cost. The cost that each entity suffers may be due to different reasons, for instance, being part of such a network requires investing in infrastructure (e.g., in the form of links), while network administrators desire to be connected via short paths to the remaining (sub)network.

Such tradeoffs, between being well-connected on the one hand and paying as less as possible on the other, have been studied under the lens of algorithmic game theory with an emphasis on understanding the network administrator objectives and analyzing their impact on the global network performance. We view each administrator as a strategic player that only cares about maximizing her utility, which, in turn, is a function of connectivity, equipment cost, and distance to other players. In other words, players make decisions according to their best interest, observe the actions of other players, and respond, if necessary, to such actions. This process eventually ends when all players are satisfied by the resulting network, i.e., no player has an incentive to unilaterally change her decision, given that all other players do not change theirs; these resulting stable networks are also called equilibria. A natural question, then, is to study how such stable networks look

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like, and how similar are they to an optimal network that maximizes a global utility function. The notion of the *price of anarchy* [15,24] is used to quantify this efficiency loss due to the selfish behavior on behalf of the network administrators, by comparing the optimal network to the worst equilibrium.

Related work: Fabrikant et al. [12] initiated the study of network formation games where the objective of each player is to be connected to the network with the minimum possible cost. Each edge has a (uniform) creation cost and the total player cost depends on the number of links that a player has to pay for, to ensure connectivity and proximity to other networks of her choice, and the sum of distances to the other players. Apart from defining the model, Fabrikant et al. also present upper and lower bounds on the price of anarchy. A series of papers improve these bounds for different ranges of the edge cost function, see e.g., [1,10,21,14]; to the best of our knowledge, the most recent results are presented in [19].

Aiming for a simpler model that would be independent of the edge cost and would allow a player to compute her best response in polynomial time, Alon et al. [3] introduced the class of basic network creation games, where no additional edges can be bought and no existing edges can be deleted. However, a player is allowed to exchange a single existing edge for a new one; this is called an *edge swap*. A player is satisfied with a graph if there is no profitable edge swap available, i.e., any such swap does not decrease the sum of distances from this player (this is the SUM version of the game) or the maximum distance from this player (this is the MAX version). A natural question, then, is how large can such graphs be, when all players are satisfied. It was shown in [3] that, in the SUM version, every swap equilibrium graph has diameter at most $2^{O(\sqrt{\log n})}$ while there exists such a swap equilibrium graph with diameter 3; the proof for the lower bound appears in [4]. Following [3], Nikoletseas et al. [22] (see also [23]) consider necessary conditions under which the diameter can be polylogarithmic for the SUM version. Furthermore, they defined a local cost network creation game where the utility of a player depends only on the degree of her neighbors. In a similar attempt to stress the impact of locality on the game, Bilò et al. [8] consider network creation games but impose restrictions on the players' knowledge about the graph. In particular, each player can observe the behavior of players that are at distance at most k from the player. Mihàlak and Schlegel [20] consider asymmetric swap equilibria where, in contrast to [3,4,22], each edge is owned by a player and each player can only swap an edge she owns, and provide upper and lower bounds on the diameter of equilibria graphs. Ehsani et al. [11] study a network formation game where each player has a fixed budget to establish edges to other players and prove bounds on the price of anarchy for both versions.

Another direction towards generalizing the model in [12] was presented by Meirom et al. [17] where players were classified in core and peripheral, in an attempt to model in a more realistic manner the actual status of Autonomous Systems on the Internet. In particular, there is a smaller group of Autonomous Systems that are well-interconnected and essentially form the Internet core, and a larger group that is connected to the Internet core via tree-like structures. Meirom et al. study in depth questions related to the price of anarchy and the price of stability for the static setting as well as to the analysis of the dynamic setting. An important special case considered in [17] is when a player may pay a neighboring node in order to reach a joint decision. In a follow-up paper, Meirom et al. [18] consider similar questions when edges may fail and, hence, each pair of nodes must be connected by at least two edge-disjoint paths. Àlvarez et al. [2] consider a setting where players sign contracts in order to exchange traffic and, among other results, provide a characterization of topologies that are pairwise stable for a given traffic matrix.

Arcaute et al. [5] as well as Lenzner [16] focus on game dynamics of network formation and creation games, while very recently, Chauhan et al. [9] studied a variant of network formation games under a setting of random edge failures. Finally, we remark that this classification of nodes into "core" and "periphery" has been also studied in different settings, see e.g., [6,7], while several papers have recently considered the price of anarchy for games played by a large number of players, see e.g., [13].

Our contribution: We first present an improved lower bound of 4 on the diameter of swap equilibria for the SUM version of basic network creation games. This is the first improvement over a lower bound of 3 due to Alon et al. [4]. We then consider network formation games with a large number of heterogeneous players and, in particular, a setting where a player may compensate another player if the total cost for both players is reduced. We present an asymptotic upper bound of 1 on the price of anarchy of such games under realistic assumptions on the cost function. Furthermore, we also show that, in general, the price of anarchy is at most 5/4 and this bound is tight, as there exists a graph with price of anarchy arbitrarily close to 5/4. Finally, we consider the case of edge failures where we require that each pair of nodes is connected via at least two edge-disjoint paths.

Roadmap: The remainder of the paper is structured as follows. We begin, in Section 2, by formally introducing the problems and by presenting the necessary definitions. Then, in Section 3, we present the improved lower bound of 4 on the diameter of sum equilibria of basic network creation games. We continue in Section 4 by considering large network formation games with heterogeneous players and monetary transfers and present improved bounds on the price of anarchy. Finally, we conclude with open problems in Section 5.

2. Preliminaries

We provide the necessary definitions used throughout the paper for basic network creation games and network formation games with heterogeneous players. In both games, the players are the nodes of a graph and each player cares only about minimizing her cost. Given a graph $G = (V, E)$, we denote by $d(u, v)$ the distance between nodes u and v in G and by $\deg(u)$ the degree of node u. In case there is no path connecting u and v in G, then we set $d(u, v) = +\infty$ and, hence, we