Doctopic: Algorithms, automata, complexity and games ARTICLE IN PRESS TCS:11438

Theoretical Computer Science ••• (••••) •••–•••

Contents lists available at ScienceDirect

Theoretical Computer Science

www.elsevier.com/locate/tcs

Strong matching preclusion number of graphs ☆

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ARTICLE INFO

Article history:
Received 31 August 2017
Received in revised form 18 November 2017
Accepted 27 December 2017
Available online xxxx
Communicated by D.-Z. Du

Keywords:
Interconnection networks
Perfect matching
Strong matching preclusion number

ABSTRACT

The *matching preclusion number* of a graph is the minimum number of edges whose deletion results in a graph that has neither perfect matchings nor almost-perfect matchings. The *strong matching preclusion number* (or simply, *SMP number*) $smp(G)$ of a graph G is the minimum number of vertices and/or edges whose deletion results in a graph that has neither perfect matchings nor almost-perfect matchings. This is an extension of the matching preclusion problem and has been introduced by Park and Ihm. In this paper, we first study the SMP number of some special graph classes, and give some sharp upper and lower bounds of SMP number. Next, graphs with large and small SMP number are characterized, respectively. In the end, we investigate the Nordhaus–Gaddum-type relations on SMP number.

1. Introduction

All graphs considered in this paper are undirected, finite and simple. We refer to the book [3] for graph theoretical notation and terminology not described here. For a graph G, let $V(G)$, $E(G)$, $e(G)$, and $\overline{G}$ denote the set of vertices, the set of edges, the size, and the complement of G, respectively. For any subset X of $V(G)$, let $G[X]$ denote the subgraph induced by X; similarly, for any subset F of $E(G)$, let $G[F]$ denote the subgraph induced by F. Let X be a set of vertices and edges. We use $G - X$ to denote the subgraph of G obtained by removing all the vertices of X together with the edges incident with them from G as well as removing all the edges of X from G. If $X = \{v\}$ and $F = \{e\}$ where v is a vertex and e is an edge, we simply write $G - v$ and $G - e$ for $G - \{v\}$ and $G - \{e\}$, respectively. For two subsets X and Y of $V(G)$ we denote by $E_G[X, Y]$ the set of edges of G with one end in X and the other end in Y. If $X = \{x\}$, we simply write $E_G[x, Y]$ for $E_G[\{x\}, Y]$. The *degree* of a vertex v in a graph G, denoted by $deg_G(v)$, is the number of edges of G incident with v. Let $\delta(G)$ and $\Delta(G)$ be the minimum degree and maximum degree of the vertices of G, respectively. The set of neighbors of a vertex v in a graph G is denoted by $N_G(v)$. The *union* $G \cup H$ of two graphs G and H is the graph with vertex set $V(G) \cup V(H)$ and edge set $E(G) \cup E(H)$. If G is the disjoint union of k copies of a graph H, we simply write $G = kH$. The *join* $G \vee H$ of two disjoint graphs G and H is the graph with vertex set $V(G) \cup V(H)$ and edge set $E(G) \cup E(H) \cup \{uv \mid u \in V(G), v \in V(H)\}$. A graph is *Hamiltonian* if it contains a Hamilton cycle. A component of a graph is *odd* or *even* according to whether it has an odd or

☆ Supported by the National Science Foundation of China (Nos. 11601254, 11551001, 11161037, and 11461054) and the Science Found of Qinghai Province (Nos. 2016-ZJ-948Q, and 2014-ZJ-907).

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https://doi.org/10.1016/j.tcs.2017.12.035
0304-3975/© 2018 Published by Elsevier B.V.

even number of vertices. A graph G is a *threshold graph*, if there exists a weight function $w : V(G) \to R$ and a real constant t such that two vertices $u, v \in V(G)$ are adjacent if and only if $w(u) + w(v) \geq t$. The *connectivity* of G, written $\kappa(G)$, is the minimum order of a vertex set $S \subseteq V(G)$ such that $G - S$ is disconnected or has only one vertex. The *edge-connectivity* of G, written $\lambda(G)$, is the minimum size of an edge-subset $X \subseteq E(G)$ such that $G - X$ is disconnected. A subset $S \subseteq V(G)$ is said to be *independent* if $E(G[S]) = \emptyset$. The *independence number* of G denoted by $\alpha(G)$ is the size of a maximum independent set in G.

A *perfect matching* in a graph is a set of edges such that every vertex is incident with exactly one edge in this set. An *almost-perfect matching* in a graph is a set of edges such that every vertex except one is incident with exactly one edge in this set, and the exceptional vertex is incident to none. So if a graph has a perfect matching, then it has an even number of vertices; if a graph has an almost-perfect matching, then it has an odd number of vertices. The *matching preclusion number* of a graph G, denoted by $mp(G)$, is the minimum number of edges whose deletion leaves the resulting graph without a perfect matching or almost-perfect matching. Any such optimal set is called an *optimal matching preclusion set*. We define $mp(G) = 0$ if G has neither a perfect matching nor an almost-perfect matching. This concept of matching preclusion was introduced in [5] and further studied in [5,14,15,6,9,12,10,11,21,24,26]. They introduced this concept as a measure of robustness in the event of edge failure in interconnection networks, as well as a theoretical connection to conditional connectivity, "changing and unchanging of invariants" and extremal graph theory. We refer the readers to [7,8] for details and additional references.

In [22], the concept of strong matching preclusion was introduced. The *strong matching preclusion number* of a graph G, denoted by $smp(G)$, is the minimum number of vertices and/or edges whose deletion leaves the resulting graph without a perfect matching or an almost-perfect matching. Any such optimal set is called an *optimal strong matching preclusion set*.

The following results are immediate.

Observation 1.1 *([22]).*

(1) *If H is a spanning subgraph of G, then $smp(H) \leq smp(G)$.*
(2) *For a graph G, $smp(G) \leq mp(G) \leq \delta(G)$.*
(3) *If e is an edge of G, then $smp(G - e) \geq smp(G) - 1$.*
(4) *If v is an edge of G, then $smp(G - v) \geq smp(G) - 1$.*

Useful distributed processor architectures offer the advantages of improved connectivity and reliability. An important component of such a distributed system is the system topology, which defines the inter-processor communication architecture. Such system topology forms the interconnection network. We refer the readers to [18] for recent progress in this area and the references in its extensive bibliography. In certain applications, every vertex requires a special partner at any given time and the matching preclusion number measures the robustness of this requirement in the event of link failures as indicated in [5]. Hence in these interconnection networks, it is desirable to have the property that the only optimal matching preclusion sets and optimal strong matching preclusion sets are those whose deletion gives an isolated vertex in the resulting graph.

Park and Ihm [22] established the SMP number and all possible minimum strong matching preclusion sets for complete graphs, regular bipartite graphs, restricted HL-graphs, and recursive circulant graphs. Park and Ihm [23] also studied the problem of strong matching preclusion under the condition that no isolated vertex is created as a result of faults, and established the conditional strong matching preclusion number for the class of restricted hypercube-like graphs, which include most nonbipartite hypercube-like networks found in the literature. SMP numbers of augmented cubes, arrangement graphs, alternating group graphs and split-star, pancake graphs, 2-matching composition networks, k-ary n-cubes are also investigated; see [4,16,13,17,27].

Let $\mathcal{G}(n)$ denote the class of simple graphs of order n. Given a graph theoretic parameter $f(G)$ and a positive integer n, the *Nordhaus–Gaddum Problem* is to determine sharp lower and upper bounds for: (1) $f(G) + f(\overline{G})$ and (2) $f(G) \cdot f(\overline{G})$, as G ranges over the class $\mathcal{G}(n)$. For some parameters, it may also be of interest to characterize such extremal graphs, that is, characterize graphs that achieve the lower bound and graphs that achieve the upper bound. The Nordhaus–Gaddum type relations have received wide investigations. Aouchiche and Hansen published a survey paper on this subject, see [1].

In Section 2, we study the SMP numbers of complete bipartite graphs, complete multipartite graphs, and threshold graphs. In term of connectivity, edge-connectivity, and independence number, sharp upper bounds of $smp(G)$ for a graph G are given in Section 3. In Section 4, we show that $0 \leq smp(G) \leq n - 1$ for a graph G of order n, and graphs with $smp(G) = 0, 1, n - 3, n - 2, n - 1$ are characterized, respectively. In the end, we investigate the Nordhaus–Gaddum-type relations on SMP number.

2. SMP number of some graph classes

From complete graphs and bipartite graphs, Park and Ihm [22] derived the following results.

Lemma 2.1 *([22]). For complete graph K_n, $smp(K_n) = n - 1$.*

Please cite this article in press as: Y. Mao et al., Strong matching preclusion number of graphs, Theoret. Comput. Sci. (2018), https://doi.org/10.1016/j.tcs.2017.12.035