Theoretical Computer Science ••• (••••) •••–•••

Contents lists available at ScienceDirect

Theoretical Computer Science

www.elsevier.com/locate/tcs

Super spanning connectivity on WK-recursive networks

Lantao You [a,b], Jianxi Fan [c], Yuejuan Han [c,*]

[a] School of Information Engineering, Suzhou Industrial Park Institute of Services Outsourcing, China
[b] Provincial Key Laboratory for Computer Information Processing Technology, Soochow University, China
[c] School of Computer Science and Technology, Soochow University, Suzhou 215006, China

ARTICLE INFO

Article history:
Received 28 March 2016
Received in revised form 13 November 2017
Accepted 18 December 2017
Available online xxxx
Communicated by C. Kaklamanis

Keywords:
WK-recursive network
Hamiltonian
Super spanning connectivity
Disjoint path cover

ABSTRACT

The WK-recursive network has received much attention due to its many favorable properties such as small diameter, large connectivity, and high degree of scalability and expandability. In this paper, we consider the super spanning connectivity properties of the WK-recursive network. We use $K(d,t)$ to denote the WK-recursive network of level t, each of which basic modules is a d-vertex complete graph, where $d > 1$ and $t \geq 1$. We prove that for any two distinct vertices μ and ν, there exist m node disjoint paths whose union covers all the vertices of $K(d,t)$ for $d \geq 4$, $t \geq 1$ and $1 \leq m \leq d-1$. Since the connectivity of $K(d,t)$ is $d-1$, the result is optimal in the worst case.

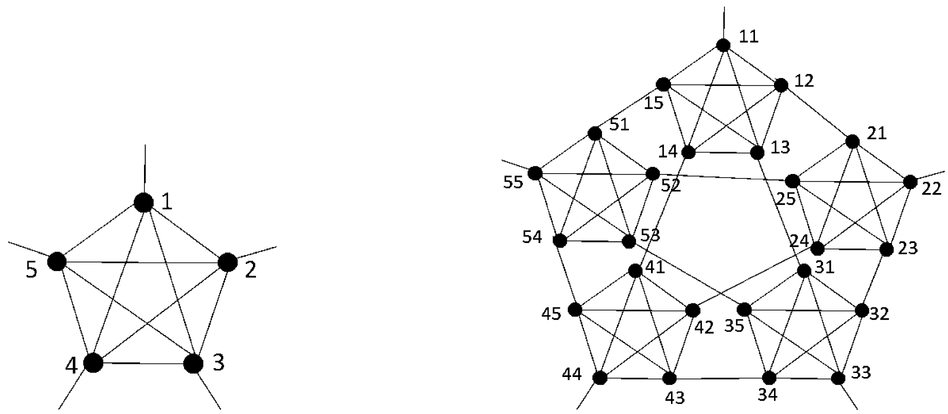
1. Introduction

In massively parallel systems, the topology plays a crucial role in issues such as communication performance, hardware costs, potentialities for efficient algorithms and fault tolerant capabilities [1]. Interconnection network topology is usually represented by a graph where vertices represent processors and edges represent links between processors. So far, interconnection networks have been widely studied [2–7]. The WK-recursive network, which was originally proposed in [8], is a class of recursively scalable interconnection networks. We use $K(d,t)$ to denote the WK-recursive network of level t, each of which basic modules is a d-vertex complete graph, where $d > 1$ and $t \geq 1$. Recently, the WK-recursive network has received much attention due to its many attractive properties, such as high degree of regularity, symmetry and efficient communication [9]. Much research has been devoted to the WK-recursive network. In [10], Fu proved that the WK-recursive network with connectivity $d-1$ is hamiltonian connected and $(d-3)$-node hamiltonian where $d \geq 4$ and $t \geq 1$ and in [11], Fu further proved that $K(d,t)$ is $(d-4)$-node hamiltonian connected for $d \geq 4$ and $t \geq 1$. In [12], Ho et al. proved $K(d,t)$ is $d-4$ fault-tolerant hamiltonian connected. In [13], Bakhshi et al. proposed one-to-one and one-to-many disjoint routings mechanisms for WK-recursive mesh networks. In [9], Fang et al. studied the pancyclic properties of the WK-recursive networks and showed that a WK-recursive network with amplitude W and level L is vertex-pancyclic for $W \geq 6$.

The construction of node-disjoint paths between a pair of distinct nodes in any network has been an important subject [14–17]. A disjoint path cover (DPC for short) of a graph is to find node-disjoint paths that cover all the vertices of the graph between two distinct vertices [18]. The node-disjoint paths are used to speed up the transfer of a large amount of data by splitting the data over several node-disjoint communication paths [19]. Additional benefits of adopting such a

* Corresponding author.
E-mail addresses: yoult@siso.edu.cn (L. You), hyj@suda.edu.cn (Y. Han).

https://doi.org/10.1016/j.tcs.2017.12.023
0304-3975/© 2017 Published by Elsevier B.V.



(a) (b)

Fig. 1. The graphs (a) $K(5,1)$ and (b) $K(5,2)$.

node-disjoint routing scheme are the enhanced robustness to node failures, and the enhanced capability of load balancing [20,21]. A k-container $C(\mu,\nu)$ of a graph G is a set of k disjoint paths between μ and ν. A k-container $C(\mu,\nu)$ of G is a k^*-container if it contains all vertices of G. A graph G is k^*-connected if there exists a k^*-container between any two distinct vertices of G. A graph G is super spanning connected if there exists a k^*-container between any two distinct vertices of G for every k with $1 \le k \le \kappa(G)$ where $\kappa(G)$ is the connectivity of G. So far, there is no work reported about the super spanning connectivity properties of $K(d,t)$.

In this paper, we study the super spanning connectivity properties for $K(d,t)$, we prove that given any two distinct vertices μ, ν of $K(d,t)$, there exist m node-disjoint paths between μ and ν whose union covers all the vertices of $K(d,t)$ where $d \ge 4$, $t \ge 1$ and $1 \le m \le d-1$. The rest of this paper is organized as follows. In Section 2, we formally define the WK-recursive network $K(d,t)$ and present some basic properties of it. In Section 3, we give the proofs of our main results. In the end, we summarize the paper in Section 4.

2. Preliminaries

In this section, we introduce some important notations and concepts. The WK-recursive network is a class of recursively scalable networks that can be constructed hierarchically by grouping basic modules. A complete graph of any size d can serve as the basic modules. In this paper, we use $K(d,t)$ to denote a WK-recursive network of level t, each of whose basic modules is a d-vertex complete graph, where $d > 1$ and $t \ge 1$.

Definition 1. Each vertex of $K(d,t)$ is labeled as a t-digit radix d number. Vertex $a_{t-1}a_{t-2}...a_1a_0$ is adjacent to (1) $a_{t-1}a_{t-2}...a_1b$, where $b \ne a_0$ and (2) $a_{t-1}a_{t-2}...a_{j+1}a_{j-1}(a_j)^{j-1}$ if $a_j \ne a_{j-1}$ and $a_{j-1} = a_{j-2} = ... = a_0$, where $(a_j)^{j-1}$ represents $j-1$ consecutive a_j. An edge incident with vertices $a_{t-1}a_{t-2}...a_1b$ and $a_{t-1}a_{t-2}...a_1c$ where $b \ne c$ is called a substituting edge. An edge incident with vertices $a_{t-1}a_{t-2}...a_{j+1}b(c)^j$ and $a_{t-1}a_{t-2}...a_{j+1}c(b)^j$ where $b \ne c$ is called a flipping edge. An edge incident to $(j)^t$ for $1 \le j \le d$ is called an open edge, which is reserved for further expansion; hence, its other end vertex is unspecified.

Each node of $K(d,t)$ is incident with $d-1$ substituting edges and one flipping edge (or open edge). The substituting edges are those within basic building blocks, and the j-flipping edges are those connecting two embedded $K(d,j)$ [11].

Definition 2. Let $c_{t-1}c_{t-2}...c_m$ be a specific $(t-m)$-digit radix d number. Define $c_{t-1}c_{t-2}...c_m \cdot K(d,m)$ as the subgraph of $K(d,m)$ induced by $\{c_{t-1}c_{t-2}...c_m a_{m-1}...a_1a_0 | a_{m-1}...a_1a_0$ is an m-digit radix d number$\}$; that is, $c_{t-1}c_{t-2}...c_m \cdot K(d,m)$, is an embedded $K(d,m)$ with the identifier $c_{t-1}c_{t-2}...c_m$.

For example, $1 \cdot K(5,1)$ is the subgraph of $K(5,2)$ induced by $\{11,12,13,14,15\}$. Let $G_m = c_{t-1}c_{t-2}...c_m \cdot K(d,m)$. If $m = t$, then $G_m = K(d,t)$. In this paper, we use $K_i(d,t-1)$ to represent the ith subgraph of level $t-1$ in $K(d,t)$ where $1 \le i \le d$ and $t \ge 2$. $K(d,t)$ consists of d copies of $K(d,t-1)$, say $K_1(d,t-1), K_2(d,t-1), ..., K_d(d,t-1)$, each subgraph has an open edge e_i of $K(d,t)$ and the edges connecting two embedded $K_i(d,t-1)$ are flipping edges where $t \ge 2$ and $1 \le i \le d$.

The open vertex set O of $K(d,t)$ is the set $\{a_{t-1}a_{t-2}...a_0 | a_i = a_{i+1}$ for $0 \le i \le t-2\}$. In other words, O contains those vertices connected with open edges. In this paper, we define $o_{(i,0)}$ to be the ith open vertex of $K(d,t)$ and $o_{(i,j)}$ to be the open vertex of subgraph $K_i(d,t-1)$ where $t \ge 2$. The open vertices of $K_i(d,t-1)$ can be labeled as $o_{(i,0)}$ and $o_{(i,j)}$ for

Please cite this article in press as: L. You et al., Super spanning connectivity on WK-recursive networks, Theoret. Comput. Sci. (2018), https://doi.org/10.1016/j.tcs.2017.12.023