# An iteration method for computing the total number of spanning trees and its applications in graph theory 

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#### Abstract

Calculating and analyzing the number of spanning trees of graphs (network models) is an important and interesting research project in wide variety of fields, such as mathematics, physics, theoretical computer science, chemistry and so on. The number of spanning trees of graphs (models) displays amounts of information on its structural features and also on some relevant dynamical properties, in particular network security, random walks and percolation. In this paper, firstly, due to lots of graphs (models) are built on the basis of various simple and small elements (components), we provide primarily some helpful network-operation, such as link-operation and merge-operation, to generate more realistic and complicated graphs (models). Secondly, considering reliability of fault-tolerance to random faults and of intrusion-tolerance to selectively remove attacks, synchronization capability and diffusion properties of networks, we present an iterative method (algorithm) for computing the total number of spanning trees. As a pellucid example, we apply our method to tree space and cycle space, notice that it is proved to be indeed a better tool. In order to reflect more widely practical meanings, we study its applications in graph theory, including ladder-graph with zero clustering coefficient, wheel-graph having nonzero clustering coefficient as constituent ingredients of maximal planar graphs. In the rest of this paper, we make a brief summary that the method described by us can be designed a program (algorithm) for obtaining easily the exact number of spanning trees of some models.


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## 1. Introduction

In the last few decades, a great number of attentions have been taken on studies of complex systems and networks in wide variety of fields, from computer science to mathematics, physics and biology science. For example, the World-Wide-Web (WWW) [1], the Internet, food webs [2], the collaboration networks [3], the web of sexual contacts [4], protein interaction networks [5], metabolic networks [6,7], the scientific literature webs and so on [8,9]. A network model can be investigated by its corresponding graph [10-12] which is a collection of vertices with edges connecting pairs of them at given mechanisms, such as random principle [13], the growth and preferential attachment presented by Barabási and Albert [14], and so forth. Besides some popular properties, scale-free feature, small-world character and community structure,

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having been deeply discovered and studied on amounts of artificial and real-life networks, some important structural and dynamical properties of network models can also be got from their panning trees.

Except for a having been researched measure (the number of walks) [15], as another important structure invariant, the number of spanning trees of a network model (graph) can only be regarded as a measure to model network security and predict this bearing capacity of fault-tolerance to random faults and of intrusion-tolerance to selective remove attacks [16]. As we know, the problem of computing the number of spanning trees of a finite graph has been solved by famous Kirchhoff's matrix-tree theorem, the product of all nonzero eigenvalues of the Laplacian matrix of the graph. But, for a huge-size graph with thousands of vertices and edges, this problem will become more intractable and tough. How to capture the accurate solutions of the number of spanning trees of models has been a demanding and interesting task, in particular on some real-world networks, and always attracts much concerns from different science fields, such as computer science, physics, mathematics, chemistry, and so on. Fortunately, there has been some useful methods, such as the theory of electrical networks, to find the precise solution for the number of spanning trees of special graph families, for instance grids, lattices, Sierpinski gaskets and Farey graph, see [17-21].

From this point, our paper is organized as follows. We will introduce some notations and definitions in Sec. 2. In Sec. 3, we debate some useful network-operation to build more complex and larger graphs (models) by a series of simpler and smaller blocks. In Sec. 4, we present a method of computing the number of spanning trees. In order to understand explicitly this method and make things more pellucid, we give two examples as application of our method to spaces of trees and cycles respectively. Based on the famous Brouwer's fixed point theorem redescribed in Sec. 5, the two graph families, ladder-graphs and wheel-graphs, are put to test in the penultimate part. In the end of this paper, we make a brief summary that our method can be designed a program (algorithm) for obtaining easily the exact solutions of number of spanning trees of some models.

## 2. Definitions

To state smoothly this article, we will use these following definitions and notations [22].

Definition 1. A spanning subgraph $G^{\prime}$ of a graph $G$ is a subgraph having the same vertex set as $G$ and a number of edges $\left|E^{\prime}\right|$ such that $\left|E^{\prime}\right| \leq|E|$. A spanning tree $T^{\prime}$ of a connected graph $G$ is a subgraph which is a tree with $\left|E\left(T^{\prime}\right)\right|=\left|V\left(T^{\prime}\right)\right|-1$. Notice that, there only is an unique path linking a vertex $u$ with another vertex $v$ in $T^{\prime}$, denoted this path as $P(u, v)$. After deleting an inner edge of $P(u, v)$ of $T^{\prime}$, the resulting graph $F^{\prime}(u, v)$ will contain two trees. That is to say, a spanning subgraph $F^{\prime}(u, v)$ of $G$ is a forest. Due to including two trees, we call the forest $F^{\prime}(u, v)$ as a $2-(u, v)$-forest of the supergraph $G$. We denote by $\chi$ and $\psi_{u v}$ the number of spanning trees $T^{\prime}$ and the number of $2-(u, v)$-forest $F^{\prime}(u, v)$ in graph $G$, respectively, and then shorted as $\Psi\left(G_{u v}\right)=\Psi\left\langle\chi, \psi_{u v}\right\rangle$.

Definition 2. We denote by $\mathcal{L}$ the set of ladder-graphs. There are two paths $P_{A}$ and $P_{B}$, respectively, with the length $l\left(P_{A}\right)$ and $l\left(P_{B}\right)$. The path $P_{A}$ contains its vertex set $V_{A}=\left\{a_{11}, a_{12}, \cdots, a_{1 i_{1}}, a_{21}, \cdots, a_{1 i_{2}}, \cdots, a_{m 1}, \cdots a_{m i_{m}}\right\}$. Similarly, the other path $P_{B}$ has its vertex set $V_{B}=\left\{b_{11}, b_{12}, \cdots, b_{1 j_{1}}, b_{21}, \cdots, b_{1 j_{2}}, \cdots, b_{m 1}, \cdots b_{m j_{m}}\right\}$. We can have

$$
\begin{equation*}
l\left(P_{A}\right)=\sum_{\alpha=1}^{m} \sum_{\beta=1}^{i_{\alpha}} a_{\alpha \beta}-1, \quad l\left(P_{B}\right)=\sum_{\alpha=1}^{m} \sum_{\beta=1}^{j_{\alpha}} b_{\alpha \beta}-1 \tag{1}
\end{equation*}
$$

For the above parallel paths $P_{A}$ and $P_{B}$, if we join $a_{11}$ with $b_{11}$ by a new path $l_{0}$ of length $\gamma_{0}$, join $a_{21}$ with $b_{21}$ by a new path $l_{1}$ of length $\gamma_{1}$, and continuously, join $a_{n i_{n}}$ with $b_{n j_{n}}$ by a new path $l_{n}$ of length $\gamma_{n}$ with $1 \leq n \leq m$. Notice that, the result leads to a generalized ladder-graph, denoted as $L\left(P_{A}, P_{B}\right)$. A generalized ladder-graph is $(p, q)$-regular if it holds $\gamma_{n}=p$ with $0 \leq n \leq m$ and $i_{1}=i_{m}=j_{1}=j_{m}=q+1, i_{\beta}=j_{\beta}=q$ with $2 \leq \beta \leq m-1$. Specially, a $(p, q)$-regular ladder-graph is $4 q$-regular if and only if $p=q$ holds true. In order to simplify discussion, let $m$ be the length of the generalized ladder-graph $L\left(P_{A}, P_{B}\right)$.

Definition 3. Let the notation $\mathcal{W}$ represent the set of wheel-graphs. For a star-graph having $n+1(n \geq 3)$ vertices, marking from the internal vertex to the last external one as $s_{0}, s_{1}, s_{2}, \cdots, s_{n}$, if we link vertex $s_{i}$ and $s_{i+1}$ with $1 \leq i \leq n-1$ by an edge and add an edge to join vertex $s_{1}$ with vertex $s_{n}$, the resulting graph can be so-named a ( $1-1-n$ )-regular wheel-graph $W(1,1, n)$. If we subdivide every edge $s_{0} s_{i}$ of $(1-1-n)$-regular wheel-graph $W(1,1, n)$ times, the generating graph will become $W(r, 1, n)$, called an $(r-1-n)$-regular wheel-graph. For the same reason, this $(1-1-n)$-regular wheel-graph $W(1,1, n)$ will become a $(1-e-n)$-regular wheel-graph $W(1, e, n)$ through subdividing every edge $s_{i} s_{i+1}$ with $1 \leq i \leq n-1$ and $s_{1} s_{n} e-1$ times. A generalized wheel-graph $W$ can be produced by subdividing every edge of the wheel-graph $W(1,1, n)$ many times, that is to say, subdivision times of all edges are various. By deleting an identifying external edge of a generalized wheel-graph $W$, we will get a corresponding damaged wheel $W^{d}$.

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