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Structured proportional representation ☆

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ABSTRACT

Multi-winner voting rules aiming at proportional representation,[1] such as those suggested by Chamberlin and Courant [2] and by Monroe [5], partition an electorate into virtual districts, such that a representative is assigned to each district; these districts are formed based on the voters' preferences. In some applications it is beneficial to require certain structural properties to be satisfied by these virtual districts. In this paper we consider situations where the voters are embedded in a network, modeled as an undirected graph, and we require the virtual districts to satisfy certain structural properties with respect to this network. Specifically, we consider two structural properties, corresponding to two different combinatorial problems: in the first problem, we require each virtual district to be connected, while in the second problem, we require the diameter of each virtual district to be small. We discuss applications of these combinatorial problems and study their computational complexity, identifying several variants and special cases which can be solved efficiently.

1. Introduction

We study a generalization of proportional representation multiwinner rules, such as those presented by Chamberlin and Courant [2] and by Monroe [5]; based on the voters' preferences, these rules select a committee of k representatives such that the following two conditions are met:

1. Each representative represents a subset of the voters.
2. Each voter is represented by exactly one representative.

Thus, in effect, the voters are partitioned into pairwise disjoint subsets, which we refer to as *virtual districts*,[2] and we assign a representative (that is, a committee member) to each such district. Ideally, each voter is represented by one of her most-desired alternatives.

☆ A Preliminary version of this work has been presented at the 16th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2017), São-Paulo, May 2017 [6]. In this long version we provide all proofs that were omitted from the conference version, improve some of the results, and discuss creating virtual districts with small diameters (whereas in the conference version we only required connectivity).

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[1] In this paper we speak about proportional representation also when we refer to the rule suggested by Chamberlin and Courant (CC); even though in some sense CC aims more at diversity (see, e.g., the book chapter by Faliszewski et al. [1]), in this paper we follow the original paper of Chamberlin and Courant [2] as well as other papers, such as the papers by Procaccia et al. [3] and by Betzler et al. [4].

[2] These are called virtual districts as they resemble electoral districts, but are based on preferences and not on geography.

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In this paper we generalize such proportional representation systems by considering not only the voters' preferences (assumed to be given as approvals; in Section 5 we discuss other possibilities), but take into account also external relations between them. That is, given an election and a network which describes relations between the voters, our goal is to partition the electorate into virtual districts, based on the voters' preferences (as specified by the election), while also requiring each virtual district to satisfy some structural properties, based on the voters' relations (as specified by the network); concretely, in this paper we consider two such structural properties, where in one problem we require each virtual district to be connected (with respect to the external network) while in another problem we require the diameter of each virtual district to be small (where an upper bound on the allowed diameter is given as an input to the algorithm). Indeed, there are other natural and interesting structural properties which we might require the districts to satisfy; in this paper we concentrate on the two structural properties described above, as two of the most basic properties of networks; in Section 5 we discuss other structural properties.

Our goal is to get the best of both worlds; that is, we want to achieve the following two goals:

1. Have our chosen committee (consisting of the representatives of all the districts) to best represent our electorate (with respect to the satisfaction of the voters from their assigned representatives).
2. Have virtual districts which conform to structural constraints: specifically, to have each district either be connected (in one problem) or to have small diameter (in another problem), according to the auxiliary network.

Formal definitions of the problems considered in this paper, called c-SPR, where the "c" stands for connectivity and d-SPR, where the "d" stands for (bounded) diameter, are given in Section 2.4.

We study the computational complexity of both c-SPR and d-SPR. It turns out that efficiently achieving our goals is not possible in general, therefore we concentrate on identifying several well-motivated tractable cases. Specifically, we model the network as a graph, and consider different graph classes, such as planar graphs, graphs with bounded degree, as well as trees and graphs with bounded treewidth.

1.1. Motivating scenarios

Below we describe several scenarios where having connected virtual districts or virtual districts of small diameter might be beneficial.

1.1.1. Political scenario

Consider a political election to be held, where a size-k committee is to be elected based on the voters' preferences, such that each voter is to be represented by a committee member. In effect, the electorate is partitioned into k virtual districts, based on the voters' representatives (and not based on geography, which is sometimes the case).

We argue that, for example, prior relationships between voters in each part might influence the effectiveness of the chosen committee. For example, since the representatives might want to discuss various issues with the voters which they represent, voters from each virtual district might periodically meet; thus, friendship relations between them might have an influence on the usefulness of such meetings. Concretely, it might be the case that each voter might be more satisfied if she knows, directly or indirectly, all the voters in her virtual district, thus having each virtual district to be a connected component with respect to the voters' friendship network might be desired. Similarly, it would be easier to conduct such meetings if the voters in each virtual district would, say, live close to each other, thus having virtual districts of small diameter would be helpful for organizing such meetings.

1.1.2. Commercial scenario

Consider a factory which can manufacture and ship to consumers at most k product types in parallel (that is, assume k production lines). The factory management might ask for the preferences of their potential customers, and choose k products which, if produced, would satisfy the highest number of people. In effect, the population of potential customers is partitioned into k shipping areas, such that each shipping area contains those potential customers which, among those k products to be produced and shipped by the factory, prefer the same product. For example, a car factory might need to select k colors for its cars, trying to maximize the number of customers which are satisfied with at least one color.

Since the factory has to ship its products to the customers, it might be desired to have each shipping area be connected, or to have each shipping of small geographical radius, with respect to a network, modelling geographic distance between the customers (possibly with some threshold, such that there is an edge between two customers if and only if the distance between them is upper-bounded by a certain threshold). The reason is that shipping the same product to several customers might be easier if those customers are close to each other; further, each product might be produced in a different geographic place, thus it is cheaper to ship only one product type at a time (say, in each truck).

1.1.3. Multiagent scenario

Consider adding hierarchy to a multiagent system, where each agent has different preferences for the agents it would like to see above it in the hierarchy. In effect, the agents are partitioned into k teams, based on their preferences, and a leader is assigned to each team.