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Optimal algorithms for inverse vertex obnoxious center location problems on graphs

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ABSTRACT

In an inverse vertex obnoxious center location problem, the aim is to modify the edge lengths at minimum total cost with respect to the modification bounds such that a predetermined vertex becomes a vertex obnoxious center location under the new edge lengths. We develop a linear time combinatorial method for the problem with edge length augmentation. For the reduction case, an algorithm with cubic running time is devised. We also show that the problem with both edge length augmentation and reduction can be solved in sextic time.

1. Introduction

In recent years, location problems become an important topic of operations research due to many practical applications in the real world. The task of a facility location problem is to locate new facilities on a system in order to serve a given set of customers in an optimal way. Two common location problems are the center problem and obnoxious (undesirable) center problem. In a center problem, one is interested in finding the best locations for establishing new desirable facilities such that the maximum of distances from the customers to the closest facility is minimized. The obnoxious center problem, however, seeks to maximize the minimum of these distances. In fact, we want to place the obnoxious facilities (like a garbage dump site, a mega-airport, a stadium or a nuclear reactor) as far away as possible from the population. In network location problems, the facilities can be placed on the vertices of a graph or in the interior of an edge. Then, the corresponding location problem is called the vertex problem or the absolute problem, respectively. For an introduction to location problems, the reader is referred to Zanjirani and Hekmatfar [19], Mirchandani and Francis [13], Cappanera et al. [8] and Plastria [17] and references therein.

This paper focuses on the inverse version of an obnoxious center location problem. In an inverse location problem, locations of the facilities are already given and one is allowed to modify some specific parameters of the problem at minimum total cost such that the prespecified facility locations become optimum. The modifying cost of an inverse location problem can be measured under different norms such as Chebyshev norm, the sum-type and bottleneck-type Hamming distances and the rectilinear norm.

Inverse approaches have been applied to several desirable/undesirable center location problems. In 1999, Cai et al. [7] showed that the inverse center location model is $\mathcal{NP}$-hard on directed general graphs. Later, Nguyen and Chassein [14]

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proved that the same problem on undirected cactus graphs is also $\mathcal{NP}$-hard. Then, the other special cases which can be solved in polynomial time have been studied on a graph $N = (V, E)$. Yang and Zhang [18] proposed an $\mathcal{O}(|V|^2 \log |V|)$ time algorithm for obtaining an optimal solution of the inverse vertex 1-center problem on unweighted trees. In 2009, Alizadeh et al. [4] developed an $\mathcal{O}(|V| \log |V|)$ time combinatorial algorithm for the inverse 1-center location problem with edge length augmentations on tree graphs. Later, Alizadeh and Burkard [2] designed an $\mathcal{O}(|V|^2)$ time algorithm for solving the inverse absolute and vertex 1-center location models on trees provided that no topology change occurs and proposed an $\mathcal{O}(|V|^2 r_v)$, $r_v < |V|$, time algorithm for the general case. For the uniform-cost inverse absolute 1-center location model on trees, the same authors designed a combinatorial algorithm with time complexity $\mathcal{O}(|V| \log |V|)$ and showed that the vertex case of this problem can also be solved in $\mathcal{O}(|V| \log |V|)$ time if all modified edge lengths remain positive [3]. In 2012, Alizadeh and Burkard in [1] considered the inverse absolute obnoxious center location problem on general graphs and developed a linear time combinatorial algorithm for it. Recently, Nguyen and Sepasian [16] investigated the inverse absolute 1-center problem on weighted trees under the Chebyshev norm and the Hamming distance. They showed that the problem is solvable in $\mathcal{O}(|V| \log |V|)$ time provided that no topology change occurs and proposed an $\mathcal{O}(|V|^2)$ time approach for the other case.

Inverse location problems are closely related to 'reverse' location problems where the task is to *improve* the given facility locations as much as possible by changing the considered parameters with respect to a given budget constraint. In 1994, Berman et al. [6] showed that the reverse 1-center location problem on unweighted graphs is $\mathcal{NP}$-hard. The same problem on unweighted trees, however, was solved in $\mathcal{O}(|V|^2 \log |V|)$ time by Zhang et al. [20]. Nguyen [15] developed an $\mathcal{O}(|V|^2)$ time method for the uniform-cost reverse 1-center problem on weighted trees. In 2016, Alizadeh and Etemad [5] proposed linear time approaches for determining an optimal solution of the reverse obnoxious center problem on general graphs under different norms. Later, Etemad and Alizadeh [10] dealt with two variants of the reverse selective center location problem on tree graphs under the Hamming and Chebyshev cost norms in which the customers are existing on a selective subset of the vertices of the underlying tree. They developed novel combinatorial algorithms with polynomial time complexities for deriving optimal solutions of the problems under investigation. Recently, the reverse selective obnoxious center location problem on cycle graphs was studied in [9] and polynomial time algorithms were proposed for it.

In this paper, we focus on the inverse version of the vertex obnoxious center location problem on general graphs in which the task is to find an optimal modification strategy of the edge lengths. We develop novel combinatorial algorithms with polynomial time complexities for obtaining optimal solutions of the problem.

This article is organized as follows: In Section 2, we give a formal problem definition of the inverse vertex obnoxious center problem and discuss some basic properties. Afterwards, Section 3 concerns an exact solution algorithm for the problem with edge length augmentation. Section 4 is dedicated to the problem with edge length reduction. Further, a solution method is presented for the problem with both edge length augmentation and reduction in Section 5. Finally, a numerical example is considered in Section 6.

2. Problem statement and preliminaries

Let an undirected connected graph $N = (V, E)$ with vertex set V and edge set E be given so that every edge $e \in E$ has a positive length $\ell(e)$. Moreover, let V be considered as the set of customer points. In a *classical vertex obnoxious center location problem* on the graph N, the aim is to find a vertex $v^* \in V$ such that the nearest neighbor to v^* is as far away as possible. An optimal solution of this problem is called a *vertex obnoxious center location* on N.

Now, let us state the *inverse vertex obnoxious center location problem* (IVOCP for short) as follows: Let the underlying graph N with associated edge lengths ℓ and customer sites V be given. Furthermore, let $s \in V$ be a predetermined vertex of N which denotes the location of an established facility. We assume without loss of generality that s does not coincide with any customer point because otherwise, the problem is trivial. We want to modify the edge lengths of N at minimum total cost such that the vertex s becomes a vertex obnoxious center of N. Let $x(e)$ and $y(e)$ denote the amounts by which the length $\ell(e)$, $e \in E$, is increased and decreased, respectively. The edge lengths of the graph N cannot be modified arbitrarily. Then, suppose that the increasing amount $x(e)$ has to obey the given upper bound $u^x(e)$ and the decreasing amount $y(e)$ has to obey the bound $u^y(e)$. Moreover, let $c(e)$ and $d(e)$ be the corresponding costs for augmenting and reducing each length $\ell(e)$ by one unit, respectively. Using the notations introduced above, the IVOCP model on the underlying graph N can formally be stated as follows:

Modify the original edge lengths $\ell(e)$, $e \in E$, to new lengths

$$\tilde{\ell}(e) = \ell(e) + x(e) - y(e)$$

such that the following three statements hold:

(i) The prespecified facility location s is a vertex obnoxious center of N under the new edge lengths $\tilde{\ell}$.
(ii) The cost function

$$\sum_{e \in E} \big(c(e)x(e) + d(e)y(e)\big)$$

is minimized.