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Randomized approximation algorithms for Planar visibility counting problem

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Abstract

Given a set S of n disjoint line segments in $\mathbb{R}^2$, the visibility counting problem (VCP) is to preprocess S such that the number of segments in S visible from any query point p can be computed quickly. This problem can be solved trivially in $O(\log n)$ query time using $O(n^4 \log n)$ preprocessing time and $O(n^4)$ space.

Gudmundsson and Morin in (2010), proposed a 2-approximation algorithm for this problem with a tradeoff between the space and the query time. For any constant $0 \leq \alpha \leq 1$, their algorithm answers any query in $O_\epsilon(m^{(1-\alpha)/2})$ time with $O_\epsilon(m^{1+\alpha})$ of preprocessing time and space, where $\epsilon > 0$ is a constant that can be made arbitrarily small and $O_\epsilon(f(n)) = O(f(n)n^\epsilon)$ and $m = O(n^2)$ is a number that depends on the configuration of the segments.

In this paper, we propose two randomized approximation algorithms for VCP. The first algorithm depends on two constants $0 \leq \beta \leq \frac{2}{3}$ and $0 < \delta \leq 1$, and the expected preprocessing time, the expected space, and the expected query time are $O(m^{2-3\beta/2} \log m)$, $O(m^{2-3\beta/2})$, and $O(\frac{1}{\delta^2} m^{\beta/2} \log m)$, respectively. The algorithm, in the preprocessing phase, selects a sequence of random samples, whose size and number depend on the tradeoff parameters. When a query point p is given by an adversary unaware of the random sample of our algorithm, it computes the number of visible segments from p, denoted by m_p, exactly, if $m_p \leq \frac{3}{\delta^2} m^{\beta/2} \log(2m)$. Otherwise, it computes an approximated value, m'_p, such that with the probability of at least $1-\frac{1}{m}$, we have $(1-\delta)m_p \leq m'_p \leq (2+2\delta)m_p$. The preprocessing time and space of the second algorithm are $O(n^2 \log n)$ and $O(n^2)$, respectively. This algorithm computes the exact value of m_p if $m_p \leq \frac{1}{\delta^2}\sqrt{n}\log n$, otherwise it returns an approximated value m''_p in expected $O(\frac{1}{\delta^2}\sqrt{n}\log n)$ time, such that with the probability at least $1 - \frac{1}{\log n}$, we have $(1-3\delta)m_p \leq m''_p \leq (1.5+3\delta)m_p$.

Keywords. computational geometry, visibility, randomized algorithm, approximation algorithm, graph theory.

1 Introduction

Problem Statement

Let $S = \{s_1, s_2, \ldots, s_n\}$ be a set of n disjoint closed line segments in the plane contained in a bounding box $\mathbb{B}$. Two points p and q in the bounding box are visible to each other with respect to S, if the open line segment $\overline{pq}$ does not intersect any segments of S. A segment $s_i \in S$ is said to be visible from a point p, if there exists a point $q \in s_i$ such that q is visible from p. *The visibility counting problem (VCP)* is to find m_p, the number of segments of S visible from a query point p. Throughout this paper, we assume that the configuration of line segments and the query point is in a general position. That is, no three end-points or the query point and two end-points are colinear.