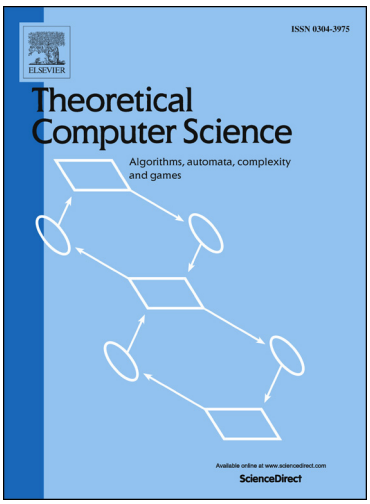

Simple and subdirectly irreducible finitely supported Cb-sets

M.M. Ebrahimi, Kh. Keshvardoost, M. Mahmoudi

PII: S0304-3975(17)30584-4
DOI: http://dx.doi.org/10.1016/j.tcs.2017.08.003
Reference: TCS 11276

To appear in: *Theoretical Computer Science*

Received date: 30 September 2016
Revised date: 30 July 2017
Accepted date: 1 August 2017

Please cite this article in press as: M.M. Ebrahimi et al., Simple and subdirectly irreducible finitely supported Cb-sets, *Theoret. Comput. Sci.* (2017), http://dx.doi.org/10.1016/j.tcs.2017.08.003

SIMPLE AND SUBDIRECTLY IRREDUCIBLE FINITELY SUPPORTED Cb-SETS

M.M. EBRAHIMI, KH. KESHVARDOOST, AND M. MAHMOUDI

Abstract. The monoid Cb of name substitutions originated by Pitts in name abstraction, and the notion of a finitely supported Cb-set appeared as an important model for semantics of programming languages in the works of Gabbay and Pitts.

The aim of this paper is to study and characterize simple, subdirectly irreducible, and indecomposable finitely supported Cb-sets. To study and find their relations, we introduce the notion of fix-simple Cb-sets, and completely describe them. Among other things, we show that under some conditions on supports, fix-simple finitely supported Cb-sets are exactly the ones whose congruence lattices are the two or three element chains. Also, it is shown that an infinite finitely supported Cb-set is simple if and only if it is a special fix-simple one. Furthermore, we observe that all the elements of a simple or subdirectly irreducible infinite Cb-set are finitely supported if we have enough elements with finite support.

1. Introduction

Let $\mathbb{D}$ be a countable infinite set. A permutation π on $\mathbb{D}$ is said to be *finitary* if it changes only a finite number of elements of $\mathbb{D}$. Consider the group $G = \mathrm{Perm_f}(\mathbb{D})$ of finitary permutations on $\mathbb{D}$, and take a set X with an action of G on it, that is, a G-set. An element $x \in X$ is said to have a *finite support* $S \subseteq \mathbb{D}$ if it is invariant (fixed) under the action of each element π of G which fixes all the elements of S (that is, if $\pi s = s$, for all $s \in S$, then $\pi x = x$).

A G-set X every element of which has a finite support is said to be a *nominal set*. The notion of a nominal set was introduced by Fraenkel in 1922, and developed by Mostowski in the 1930s under the name of legal sets. The legal sets were used to prove the independence of the axiom of choice from the other axioms (in the classical Zermelo-Fraenkel (ZF) set theory).

2010 *Mathematics Subject Classification.* 08A30, 08B26, 20B30, 20M30, 20M35, 68Q55.

Key words and phrases. Cb-set, support, simple, subdirectly irreducible, indecomposable.