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Formalization of Fractional Order PD Control Systems in HOL4

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Abstract: Higher-order logic theorem proving method is applied to analyze fractional order PD control systems in this paper. Theorem proving is based on rigorous logic and correct mathematics theory. Firstly, existent conditions and formal model of fractional calculus Caputo definition is established in higher order logic theorem prover. Then some properties are verified, including homogeneity, linearity property, fractional differential of constant and relationship between integer order differential and fractional differential. And formalization of Laplace transform based on fractional calculus Caputo definition is given and we apply it to illustrate advantages of fractional calculus Caputo definition. Initial values based on fractional calculus Caputo definition have exact physical meanings. And then formal modeling of fractional order PD controller is verified based on the above properties. We also verify the stability and other properties of fractional order PD controller by theorem proving method. Lastly, steady-state error is analyzed based on formalization of fractional calculus Caputo definition. It shows that fractional order PD controller can achieve a better control performance than integer order PD controller.

Key words: Fractional Calculus; Caputo Definition; Higher Order Logic; Theorem Proving; Fractional Order PD Controller.

1 Introduction

The essence of nature is fractional order [1]. Fractional models can accurately describe the essential characteristics and behaviors of real systems [2]. Fractional order control systems can make systems obtain the ideal control performance [3]. Fractional order PD controller is a kind of fractional order controllers, which is proposed in 1994[4]. Compared with traditional PD controller, fractional order PD controller has one more degree of freedom. That is the differential order, which makes parameter tuning scope larger and realizes more accurate control effect. Correct analyses of fractional control system characteristics will be conducive to the application of fractional order controllers in practice.

Fractional order control systems are based on fractional calculus. Traditional manual analysis is very prone to error because of the complexity of the required calculations. The most commonly used analytical technique for fractional order systems is the simulation method, which is based on numerical computation. However, analytical solutions have only been found with a few specific fractional order systems. Most fractional order systems yield approximate results with a minimum error using an optimal algorithm. A simulation method cannot procure accurate results. As a result, such systems are likely to contain bugs. In addition, fractional calculus relates to infinite series summation. The simulation for infinite series summation requires a huge amount of memory. Theorem proving provides a way to formally analyze fractional order PD control systems. Theorem proving is one kind of formal verification methods. Theorem proving method converts mathematical model into logical model of fractional order systems and their properties. It deduces the correctness of properties from logical rules. And it is the strictest and most standardized method so far and the credibility of verified conclusion is also the highest. The tool we use in this paper is higher-order logic theorem prover which is abbreviated as HOL4.

Fractional calculus has three commonly used definitions. They are fractional calculus Grünwald-Letnikov (GL) definition, fractional calculus Riemann-Liouville (RL) definition and fractional calculus Caputo definition. Fractional calculus RL and Caputo definitions are the improvement of fractional calculus GL definition. And fractional calculus Caputo definition is more suitable for engineering practices. Fractional calculus Caputo definition is bounded for the derivative of constant. Moreover, it can make Laplace transform more concise and its initial values are given in the form of integer order which has a reasonable physical explanation. These advantages make fractional calculus Caputo definition more useful than other definitions. In this paper, formalizations of fractional calculus Caputo definition and fractional order PD controller are proposed.

The rest of the paper is organized as follows: we present a research framework in Section 2. Descriptions and formalizations of fractional calculus Caputo definition and its classical properties are verified in Section 3. Fractional order PD control systems are