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The multi-service center problem ☆

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ABSTRACT

We propose a new type of network location problem for placing multiple but distinct facilities, called the *p-service center* problem. In this problem, we are to locate p facilities in the graph, each of which provides distinct service required by all vertices. For each vertex, its *p-service distance* is the summation of its weighted distances to the p facilities. The objective is to minimize the maximum value among the p-service distances of all vertices. In this paper, we show that the p-service center problem on a general graph is NP-hard, and propose a simple approximation algorithm of factor p/c for any integer constant c. Moreover, we study the basic case $p = 2$ on paths and trees. When the underlying network is a path, we solve the 2-service center problem in $O(n)$ time, where n is the number of vertices. When the underlying network is a tree, we give an $O(n^3)$-time algorithm for the case of nonnegative weights, an $O(n \log n)$-time algorithm for the case of positive weights, and an $O(n)$-time algorithm for the case of uniform weights.

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1. Introduction

Optimally choosing location of facilities on networks is a practical and significant problem in our daily life. Thus, network location problems have received much attention from researchers in the fields of transportation and communication over four decades [2–4,6–10,12–14]. Traditionally, network location problems consider only identical facilities while determining the optimal location of multiple facilities, and each client will thus be served by its closest facility. For example, the well-known *p-center* problem is to place p identical facilities in the network such that the maximum distance from each client to its closest facility is minimized, and the *p-median* problem is to place p identical facilities such that the total distance from all clients to their corresponding closest facilities is minimized.

However, when an organization seeks for providing large-scale and integrated services, it is possible that each single facility to be located cannot afford to provide all kinds of services due to the complexity and cost, and thus they have to play different roles in the whole picture and offer dissimilar but interrelated services. Take the freight traffic of logistics centers as an example. We want to set up several logistics centers in the transportation network to provide many kinds of goods, but various factors make it difficult to store all kinds of goods in each single logistics center, such as places of production, storage cost, market demand, and so on. It is thus more efficient for each logistics center to store only limited kinds of goods, and together they still meet all kinds of requirements from everywhere in the network. As another example, while planning the layout of a new town, police stations, fire stations, and hospitals play distinct but closely-related roles, but the town environment and population distribution may force them to be located separately. Under such circumstances,

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setting up each facility in an individual and independent way may fail to optimize the quality of service, and comprehensive planning is therefore required for integrating their services. It follows that the optimal placement of such distinct but cooperative facilities will be very different from that of identical facilities. We are interested in this new kind of network location problems, and call them the *multi-service location* problems.

In this paper, we define the first problem of this kind, the p-*service center* problem. There are p facilities to be placed in the network, each of which provides one distinct kind of service. Each client in the network has its own demand level for each kind of service, and how well the client is served is measured by its p-*service distance*, the total transportation cost from the client to the p facilities. The objective of the p-service center problem is to find a placement of the p facilities that minimizes the maximum value among the p-service distances of all clients.

We study the p-service center problem and obtain several approximate and polynomial-time algorithms for different cases. When the underlying graph is a general graph, we show that the p-service center problem for general p is NP-hard, and give a simple approximation algorithm of factor p/c, where c is a small integer constant. Then, for the basic case $p = 2$, we propose polynomial-time solutions on path graphs and tree graphs. When the underlying graph is a path, we convert the 2-service center problem into a linear program and solve it in $O(n)$ time, where n is the number of vertices. For tree graphs with nonnegative vertex weights, we extend the linear programming concept for paths to give an $O(n^3)$ solution. For the case that all vertex weights are positive numbers, we develop a novel prune-and-search strategy, called the *branch trimming* approach, which solves the problem in $O(n \log n)$ time. If the vertex weights are uniform (all equal to 1), the time complexity can be further reduced to $O(n)$ time.

Recently, Anzai et al. [1] showed that the p-service center problem is NP-hard even for split graphs with identical edge lengths. Except that, there is no existing result for the p-service center problem in the literature, and we list some results about the p-center problem here for reference. When the underlying network is a general graph, Kariv and Hakimi [8] proved that the p-center problem is NP-hard. Moreover, Plesník [12] showed that it is impossible to obtain a solution within a ratio of $\delta < 2$ unless P = NP, and provided a 2-approximation algorithm for this problem with triangle inequality [13]. When the underlying network is a tree, Fredrickson [4] proposed an $O(n)$-time algorithm for the vertex-unweighted case, and Cole [3] gave an $O(n \log^2 n)$-time algorithm for the weighted case.

The cases in which p is a small constant have also been extensively studied in the literature. For $p = 1$, Kariv and Hakimi [8] studied the 1-center problem on a general graph, and provided an $O(mn \log n)$-time algorithm for the weighted case and an $O(mn + n \log n)$-time algorithm for the unweighted case, where m is the number of edges. Megiddo [10] proposed a linear-time algorithm for finding the weighted 1-center in a tree. Then, Ben-Moshe et al. [2] gave a linear time algorithm for finding the weighted 2-center in a tree and an $O(n \log n)$-time algorithm for finding the weighted 3-center and 4-center in a tree.

The rest of this paper is organized as follows. Section 2 gives formal definitions and basic properties. In Section 3, we study the p-service center problem on general graphs. Then, in Section 4, we discuss the case that $p = 2$ on paths and trees. Finally, in Section 5, we conclude the paper.

2. Notation and preliminaries

Let $p \geq 1$ be an integer, and $G = (V, E, p, W)$ be an undirected connected graph, where $V = \{1, 2, \ldots, n\}$ is the vertex set, E is the edge set, p is an integer, and W is an n-by-p matrix of nonnegative real numbers $[w_{i,j}]_{n \times p}$. Each vertex $i \in V$ is associated with p weights $w_{i,1}, w_{i,2}, \ldots, w_{i,p}$, which correspond to the p elements in the i-th row of W. Each edge $e \in E$ has a nonnegative length and is assumed to be rectifiable. Also, G denotes the continuum set of all points of the graph, so that the notation $x \in G$ means that x is a point along any edge of G, which may or may not be a vertex of G. For any two points $x, y \in G$, let $P(x, y)$ be the shortest path between x and y, and $d(x, y)$ be the total length of $P(x, y)$. Suppose that the matrix of shortest distances between all pairs of vertices of G is given.

A p-*location* is a tuple $(x_1, x_2, \ldots, x_p)$ of p points in G (not necessarily distinct). Given an arbitrary p-location $X = (x_1, x_2, \ldots, x_p)$, the p-*service distance* from a vertex $i \in V$ to X is defined to be

$$D(i, X) = \sum_{1 \leq j \leq p} w_{i,j} \times d(i, x_j),$$

and the p-*service cost* of X is defined to be

$$F(X) = \max_{i \in V} D(i, X).$$

The p-*service center* problem is to find a p-location X^*, called a p-*service center*, that minimizes $F(X^*)$.

Note that, in the case that $p = 1$, the p-service center problem is equivalent to the p-center problem. Also, in the p-center problem, it is generally assumed that $p < n$, since the case $p \geq n$ implies trivial solutions of zero maximum distance. However, in the p-service center problem, p is irrelevant to n.

A p-*family* is a tuple $(H_1, H_2, \ldots, H_p)$ of p subgraphs of G (not necessarily distinct). For convenience, given a p-location $X = (x_1, x_2, \ldots, x_p)$ and a p-family $(H_1, H_2, \ldots, H_p)$, we write $X \in (H_1, H_2, \ldots, H_p)$ to mean that $x_j \in H_j$ for $1 \leq j \leq p$, and $X \notin (H_1, H_2, \ldots, H_p)$ if there exists at least one index j such that $x_j \notin H_j$. For a p-family $(H_1, H_2, \ldots, H_p)$, a p-location $X \in (H_1, H_2, \ldots, H_p)$ is called an $(H_1, H_2, \ldots, H_p)$-*optimal solution* if $F(X) = \min_{X' \in (H_1, H_2, \ldots, H_p)} F(X')$. For simplicity, let

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