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A width parameter useful for chordal and co-comparability graphs [☆]

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ABSTRACT

Belmonte and Vatshelle (TCS 2013) used mim-width, a graph width parameter bounded on interval graphs and permutation graphs, to explain existing algorithms for many domination-type problems on those graph classes. We investigate new graph classes of bounded mim-width, strictly extending interval graphs and permutation graphs. The graphs $K_t \boxminus K_t$ and $K_t \boxminus S_t$ are graphs obtained from the disjoint union of two cliques of size t, and one clique of size t and one independent set of size t respectively, by adding a perfect matching. We prove that:

- interval graphs are $(K_3 \boxminus S_3)$-free chordal graphs; and $(K_t \boxminus S_t)$-free chordal graphs have mim-width at most $t-1$,
- permutation graphs are $(K_3 \boxminus K_3)$-free co-comparability graphs; and $(K_t \boxminus K_t)$-free co-comparability graphs have mim-width at most $t-1$,
- chordal graphs and co-comparability graphs have unbounded mim-width in general.

We obtain several algorithmic consequences; for instance, while Minimum Dominating Set is NP-complete on chordal graphs, it can be solved in time $n^{\mathcal{O}(t)}$ on $(K_t \boxminus S_t)$-free chordal graphs. The third statement strengthens a result of Belmonte and Vatshelle stating that either those classes do not have constant mim-width or a decomposition with constant mim-width cannot be computed in polynomial time unless $P = NP$.
We generalize these ideas to bigger graph classes. We introduce a new width parameter *sim-width*, of stronger modeling power than mim-width, by making a small change in the definition of mim-width. We prove that chordal graphs and co-comparability graphs have sim-width at most 1. We investigate a way to bound mim-width for graphs of bounded sim-width by excluding $K_t \boxminus K_t$ and $K_t \boxminus S_t$ as induced minors or induced subgraphs, and give algorithmic consequences. Lastly, we show that circle graphs have unbounded sim-width, and thus also unbounded mim-width.

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Fig. 1. $K_5 \boxminus S_5$ and $K_5 \boxminus K_5$.

1. Introduction

The study of structural graph "width" parameters like tree-width and clique-width have been ongoing since at least the 1990s, and their algorithmic use has been steadily increasing [9]. A parallel and somewhat older field of study gives algorithms for special graph classes, such as chordal graphs and permutation graphs. Since the introduction of algorithms based on tree-width, which generalized algorithms for trees and series-parallel graphs, these two fields have been connected. Recently the fields became even more intertwined with the introduction of mim-width in 2012 [32], which yielded generalized algorithms for quite large graph classes. In the context of parameterized complexity a negative relationship holds between the modeling power of graph width parameters, i.e. what graph classes have bounded parameter values, and their analytical power, i.e. what problems become FPT or XP. For example, the family of graph classes of bounded width is strictly larger for clique-width than for tree-width, while under standard complexity assumptions the class of problems solvable in FPT time on a decomposition of bounded width is strictly larger for tree-width than for clique-width. For a parameter like mim-width its algorithmic use must therefore be carefully evaluated against its modeling power, which is much stronger than clique-width. A common framework for defining graph width parameters is by branch decompositions over the vertex set. This is a natural hierarchical clustering of a graph G, represented as an unrooted binary tree T with the vertices of G at its leaves. Any edge of the tree defines a cut of G given by the leaves of the two subtrees that result from removing the edge from T. Judiciously choosing a cut-function to measure the complexity of such cuts, or rather of the bipartite subgraphs of G given by the edges crossing the cuts, this framework then defines a graph width parameter by a minmax relation, minimum over all trees and maximum over all its cuts. Restricting T to a path with added leaves we get a linear variant. The first graph parameter defined this way was carving-width [29], whose cut-function is simply the number of edges crossing the cut, and whose modeling power is weaker than tree-width. The carving-width of a graph G is thus the minimum, over all such branch decomposition trees, of the maximum number of edges crossing a cut given by an edge of the tree. Several graph width parameters have been defined this way. For example the mm-width [32], whose cut function is the size of the maximum matching and has modeling power equal to tree-width; and the rank-width [25], whose cut function is the GF[2]-rank of the adjacency matrix and has modeling power equal to clique-width.

This framework was used by Vatshelle in 2012 [32] to define the parameter mim-width whose cut function is the size of the maximum induced matching of the graph crossing the cut. Note that low mim-width allows quite complex cuts. Carving-width one allows just a single edge, mm-width one a star graph, and rank-width one a complete bipartite graph with some isolated vertices. In contrast, mim-width one allows any cut where the neighborhoods of the vertices in a color class can be ordered linearly w.r.t. inclusion. The modeling power of mim-width is much stronger than clique-width. Belmonte and Vatshelle showed that interval graphs and permutation graphs have linear mim-width at most one, and circular-arc graphs and trapezoid graphs have linear mim-width at most two [3],[3] whereas the clique-width of such graphs can be proportional to the square root of the number of vertices. With such strong modeling power it is clear that the analytical power of mim-width must suffer. The cuts in a decomposition of constant mim-width are too complex to allow FPT algorithms for interesting NP-hard problems. Instead, what we can get is XP algorithms, for the class of LC-VSVP problems [8] – locally checkable vertex subset and vertex partitioning problems – defined in Section 2.4. For classes of bounded mim-width this gives a common explanation for many classical results in the field of algorithms on special graph classes.

In this paper we extend these results on mim-width in several ways. Firstly, we show that chordal graphs and co-comparability graphs have unbounded mim-width, thereby answering a question by Belmonte and Vatshelle [3], and giving evidence to the intuition that mim-width is useful for large graph classes having a linear structure rather than those having a tree-like structure. Secondly, by excluding certain subgraphs, like $K_t \boxminus S_t$ obtained from the disjoint union of a clique of size t and an independent set of size t by adding a perfect matching, we find subclasses of chordal graphs and co-comparability graphs for which we can nevertheless compute a bounded mim-width decomposition in polynomial time and thereby solve LC-VSVP problems. See Fig. 1 for illustrations of $K_t \boxminus S_t$. Note that already for $K_3 \boxminus S_3$-free chordal graphs the tree-like structure allowed by mim-width is necessary, as they have unbounded linear mim-width. Thirdly, we introduce sim-width, of modeling power even stronger than mim-width, and start a study of its properties.

The graph width parameter sim-width is defined within the same framework as mim-width with only a slight change to its cut function, simply requiring that a special induced matching across a cut cannot contain edges between any pair of vertices on the same side of the cut. This exploits the fact that a cut function for branch decompositions over the vertex set of a graph need not be a parameter of the bipartite graph on edges crossing the cut. The cuts allowed by sim-width are

[3] In [3], all the related results are stated in terms of d-neighborhood equivalence, but in the proof, they actually gave a bound on mim-width or linear mim-width.