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Signal set tissue systems and overlapping localities

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ABSTRACT

In signal set tissue systems with overlapping localities (ssolt-systems) evolution rules can be associated with several cells. Rules may influence each other through instant signalling and, moreover, they are synchronised when sharing activated cells. The latter is a new feature. We study the behaviour of ssolt-systems in the form of labelled step transition systems. This allows to compare ssolt-systems with and without signalling and overlapping localities. Next the synthesis problem is considered, i.e., the question when given a step transition system, how to effectively construct an ssolt-system exhibiting this behaviour. To this end, ssolt-systems are related to a new class of Petri nets, that are behaviourally equivalent to ssolt-systems. It is shown how certain region based synthesis techniques can be applied to these nets and hence are also available for ssolt-systems.

1. Introduction

Tissue systems are a computational model inspired by the way biochemical reactions and interactions take place inside and among living cells ([21,22]). Like membrane systems, tissue systems are defined in terms of evolution rules that specify how objects (molecules) can be combined to form new objects. In both types of systems reactions are localised, i.e., they take place in compartments where their reactants reside and their products can either remain in the originating compartment or be delivered to a neighbouring one. A membrane system has an associated hierarchical structure (a tree) that reflects the structure of a cell; the tree corresponds to the nesting of its compartments separated by membranes. The more general tissue systems use a graph to formalise neighbourhood relations between compartments, now like cells arranged in a tissue. Motivated by reaction systems [6,8,9] that model biochemical processes using a qualitative rather than a quantitative approach, [12] has introduced a variant of membrane systems — set membrane systems, independently introduced also in [1] — with evolution rules that no longer refer to multisets of objects (indicating how many instances of each object are involved), but rather to sets (merely indicating presence/absence) of objects. In [13] set tissue systems, i.e., tissue systems with such qualitative evolution rules, are enhanced with instant signalling that makes it possible to fast process certain objects even within the same computational step that has produced them. This extension is an abstraction of cell signalling, a complex system of communication employed in cellular biochemical processes. In a biological micro environment, signalling is a fundamental feature, governing and coordinating the activities of cells. This ability to pass on and respond immediately to the presence of signalling objects has been abstracted in [13] in the framework of set tissue systems through the con-

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current execution of evolution rules based on a local maximality (eagerness) principle that reflects that everything that can happen (in an active compartment) does not wait and occurs as soon as possible. As said, the various models of membrane and tissue systems discussed so far are based on the idea that reactions take place locally, i.e., in individual compartments. Moreover, executions in different compartments are unrelated.

In this paper, we introduce evolution rules that are associated with conglomerates of compartments, i.e., each rule has a *locality* consisting of all compartments where it may occur. The underlying idea is that not only the reactions taking place within and among cells are influencing each other by the production of biochemical substances, but that there may also be external stimuli (mechanical, temperature, light etc.) that simultaneously affect several cells within a tissue. A rule that is associated with thus 'activated' compartments will then be simultaneously executed in each of the compartments with which it is associated. In this paper we are interested in the behaviour of the resulting *signal set tissue system with overlapping localities*, or SSOLT-systems. To capture this behaviour, labelled transition systems (so-called *concurrent reachability graphs*) are used that describe how starting from an initial configuration (state), a SSOLT-system evolves from configuration to configuration through a 'locally maximal' execution of evolution rules. Here, we distinguish between two types of semantics depending on the interpretation of local maximality, i.e., the interpretation of the eagerness underlying the locally maximal execution of the enabled rules. In the first interpretation, a rule is involved whenever *all* its compartments are activated (have resources). In the second semantics, a rule is involved whenever at least one (*any*) of its compartments is activated. Consequently, there will be two kinds of concurrent reachability graphs that can be defined by a SSOLT-system. As will be shown, in both these semantics, signalling and simultaneous activation of compartments have each their own effect on the expressiveness of the resulting models in terms of the generated reachability graphs.

On the other hand, we will also consider how to obtain from a labelled step transition system a SSOLT-system that behaves as specified, i.e., has a concurrent reachability graph (of type *all* or *any*) that is isomorphic to the given transition system — if at all possible. Algorithmic methods to synthesise concurrent systems from behavioural specifications (like step transition systems) provide an attractive way to construct systems that behave according to specification. This so-called synthesis problem has been extensively studied in the context of Petri nets, a well-established mathematical framework for the modelling of concurrent and distributed systems (see, e.g., [23,24]). As we will demonstrate, some of the theory developed there can be extended and then transferred to SSOLT-systems.

Thus, we will relate the concurrent reachability graphs of SSOLT-systems to those of a suitable class of Petri nets. At a very basic level, the dynamics of membrane and tissue systems is similar to that of Petri nets (see [10]). Molecules in a compartment correspond to tokens in a specific place and evolution rules can be viewed as transitions related to input and output places in accordance with the objects consumed and produced by the rule. Once a similar mode of execution for both models has been agreed upon, there is a one-to-one correspondence between configurations and markings (token distributions) and between the concerted execution of multiple rules and the simultaneous firing of their related transitions, guaranteeing isomorphic reachability graphs. This correspondence has been proven to be robust in the sense that various concepts and methods could be transferred from one framework to the other. From the observation that the evolution rules of a membrane or tissue system are localised (belong to compartments) came the idea to also associate a *locality* with Petri net transitions and to define a locally maximal execution mode ([18,17]). And membrane systems with qualitative evolution rules relate to set nets — that have a firing rule based on sets rather than multisets — with localities ([12]). Instant signalling on the other hand involves an application of the special so-called a/sync places that can be used for the instantaneous transfer of tokens from an input transition to an output transition ([11]). Overlapping localities of transitions, resembling the overlap of localities of the rules in a SSOLT-system, have been considered in [20].

Our solution to the synthesis problem will be based on the notion of *region*, introduced in the seminal paper [7] for the synthesis of Elementary Net systems with a sequential execution semantics, a basic class of Petri Nets. Over the years, this idea has been further developed and extended in various directions, including synthesis modules of tool kits, various application areas of other net classes, other execution semantics, and different behavioural specification models. One of the key advances in the design of region based solutions has been the development in [3] of a general approach for dealing with region based synthesis for a range of synthesis problems. This approach is founded on so-called τ-nets and τ-regions. The parameter τ conveniently captures the marking information and different connections between places and transitions of varying classes of Petri nets, removing the need to re-state and re-prove the basic results every time a net model is modified. Once a class of Petri nets has been shown to be a class of τ-nets, i.e., to correspond to a class of τ-nets for some suitable τ, this general method can be applied.

Based on the above considerations, we thus introduce a new class of Petri nets, so-called signal set nets with overlapping localities, or SSOL-nets. It is demonstrated that indeed SSOLT-systems and SSOL-nets closely correspond to each other both in structure and dynamics and hence for each of the two execution modes (*all* and *any*), the related concurrent reachability graphs are essentially the same. Finally, we show that SSOL-nets are indeed a class of τ-nets. Hence it is possible to address the synthesis problem using τ-regions. However, since SSOL-nets have two kinds of places (normal ones and signal places), in this case the parameter τ is based on two net-types to describe the connections between places and transitions, which is a new feature.

The paper is organized as follows. After a preliminary section, SSOLT-systems are introduced together with their *all* and *any* execution semantics in Section 3. In the same section, the expressive power of the different features is compared in terms of the concurrent reachability graphs. Section 4 introduces SSOL-nets and relates them to SSOLT-systems, whereas it is