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More results on weighted independent domination ☆

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ABSTRACT

Weighted independent domination is an NP-hard graph problem, which remains computationally intractable in many restricted graph classes. In particular, the problem is NP-hard in the classes of sat-graphs and chordal graphs. We strengthen these results by showing that the problem is NP-hard in a proper subclass of the intersection of sat-graphs and chordal graphs. On the other hand, we identify two new classes of graphs where the problem admits polynomial-time solutions.

1. Introduction

Independent domination is the problem of finding in a graph an inclusionwise maximal independent set of minimum cardinality. This is one of the hardest problems of combinatorial optimisation and it remains difficult under substantial restrictions. In particular, it is NP-hard for so-called sat-graphs, where the problem is equivalent to satisfiability [19]. It is also NP-hard for planar graphs, triangle-free graphs, graphs of vertex degree at most 3 [3], line graphs [18], chordal bipartite graphs [7], etc.

The weighted version of the problem (abbreviated WID) deals with vertex-weighted graphs and asks to find an inclusionwise maximal independent set of minimum total weight. This version is provenly harder, as it remains NP-hard even for chordal graphs [5], where independent domination can be solved in polynomial time [8]. In the present paper, we strengthen two NP-hardness results by showing that WID is NP-hard in a proper subclass of the intersection of sat-graphs and chordal graphs.

On the positive side, it is known that the problem is polynomial-time solvable for interval graphs, permutation graphs [4], graphs of bounded clique-width [6], etc.

Let us observe that all classes mention above are hereditary, i.e. closed under taking induced subgraphs. It is well-known (and not difficult to see) that a class of graphs is hereditary if and only if it can be characterised in terms of minimal forbidden induced subgraphs. Unfortunately, not much is known about efficient solutions for the WID problem on graph classes defined by *finitely many* forbidden induced subgraphs. Among rare examples of this type, let us mention cographs and split graphs.

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- A *cograph* is a graph in which every induced subgraph with at least two vertices is either disconnected or the complement of a disconnected graph. The cographs are precisely P_4-free graphs, i.e. graphs containing no induced P_4. In the case of cographs, the problem can be solved efficiently by means of modular decomposition.
- A *split graph* is a graph whose vertices can be partitioned into a clique and an independent set. In terms of forbidden induced subgraphs, the split graphs are the graphs which are free of $2K_2, C_4$ and C_5. The only available way to solve WID efficiently for a split graph is to examine all its inclusionwise maximal independent sets, of which there are polynomially many.

The class of sat-graphs, mentioned earlier, consists of graphs whose vertices can be partitioned into a clique and a graph of vertex degree at most 1. Therefore, sat-graphs form an extension of split graphs. With this extension the complexity status of the problem jumps from polynomial-time solvability to NP-hardness. In the present paper, we study two other extensions of split graphs and show polynomial-time solvability in both of them.

The first of them deals with the class of $(P_5, \overline{P_5})$-free graphs, which also extends the cographs. From an algorithmic point of view, this extension is resistant to any available technique. To crack the puzzle for $(P_5, \overline{P_5})$-free graphs, we develop a new decomposition scheme combining several algorithmic tools. This enables us to show that the WID problem can be solved for $(P_5, \overline{P_5})$-free graphs in polynomial time.

The second extension of split graphs studied in this paper deals with the class of $(P_5, \overline{P_3 + P_2})$-free graphs. To solve the problem in this case, we develop a tricky reduction allowing us to reduce the problem to the first class.

Let us emphasise that in both cases the presence of P_5 among the forbidden graphs is necessary, because each of $\overline{P_5}$ and $\overline{P_3 + P_2}$ contains a C_4 and by forbidding C_4 alone we obtain a class where the problem is NP-hard [3]. Whether the presence of P_5 among the forbidden graphs is sufficient for polynomial-time solvability of WID is a big open question. For the related problem of finding a maximum weight independent set (WIS), this question was answered only recently [12] after several decades of attacking the problem on subclasses of P_5-free graphs (see e.g. [2,9,11]). In particular, prior to solving the problem for P_5-free graphs, it was solved for (P_5, H)-free graphs for all graphs H with at most 5 vertices, except for $H = C_5$.

WID is a more stubborn problem, as it remains NP-hard in many classes where WIS can be solved in polynomial time, such as line graphs, chordal graphs, bipartite graphs, etc. In [13], the problem was solved in polynomial time for many subclasses of P_5-free graphs, including (P_5, H)-free graphs for all graphs H with at most 5 vertices, except for $H = \overline{P_5}$, $H = \overline{P_3 + P_2}$ and $H = C_5$. In the present paper, we solve the first two of them, leaving the case of (P_5, C_5)-free graphs open. We believe that WID in (P_5, C_5)-free graphs is polynomially equivalent to WID in P_5-free graphs. Determining the complexity status of the problem in both classes is a challenging open question. We discuss this and related open questions in the concluding section of the paper.

The rest of the paper is organised as follows. In the remainder of the present section, we introduce basic terminology and notation. In Section 3 we solve the problem for $(P_5, \overline{P_5})$-free graphs, and in Section 4 we solve it for $(P_5, \overline{P_3 + P_2})$-free graphs.

All graphs in this paper are finite, undirected, without loops and multiple edges. The vertex set and the edge set of a graph G are denoted by $V(G)$ and $E(G)$, respectively. A subset $S \subseteq V(G)$ is

- *independent* if no two vertices of S are adjacent,
- a *clique* if every two vertices of S are adjacent,
- *dominating* if every vertex not in S is adjacent to a vertex in S.

For a vertex-weighted graph G with a weight function w, by $id_w(G)$ we denote the minimum weight of an independent dominating set in G.

If v is a vertex of G, then $N(v)$ is the *neighbourhood* of v (i.e. the set of vertices adjacent to v) and $V(G) \setminus N(v)$ is the *antineighbourhood* of v. We say that v is *simplicial* if its neighbourhood is a clique, and v is *antisimplicial* if its antineighbourhood is an independent set.

Let S be a subset of $V(G)$. We say that a vertex $v \in V(G) \setminus S$ *dominates* S if $S \subseteq N(v)$. Also, v *distinguishes* S if v has both a neighbour and a non-neighbour in S. By $G[S]$ we denote the subgraph of G induced by S and by $G - S$ the subgraph $G[V \setminus S]$. If S consists of a single element, say $S = \{v\}$, we write $G - v$, omitting the brackets.

If G is a connected graph but $G - S$ is not, then S is a *separator* (also known as a cut-set). A *clique separator* is a separator which is also a clique.

As usual, P_n, C_n and K_n denote a chordless path, a chordless cycle and a complete graph on n vertices, respectively. Given two graphs G and H, we denote by $G + H$ the disjoint union of G and H, and by mG the disjoint union of m copies of G.

We say that a graph G contains a graph H as an induced subgraph if H is isomorphic to an induced subgraph of G. Otherwise, G is H-free.

A class $\mathcal{Z}$ of graphs is hereditary if it is closed under taking induced subgraphs, i.e. if $G \in \mathcal{Z}$ implies that every induced subgraph of G belongs to $\mathcal{Z}$. It is well-known that $\mathcal{Z}$ is hereditary if and only if graphs in G do not contain induced subgraphs from a set M, in which case we say that M is the set of forbidden induced subgraphs for $\mathcal{Z}$.

For an initial segment of natural numbers $\{1, 2, \ldots, n\}$ we will often use the notation $[n]$.