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Sampling Weighted Perfect Matchings on the Square-Octagon Lattice

Prateek Bhakta * Dana Randall †

Abstract

Perfect matchings of the square-octagon lattice, also known as "fortresses" [19], have been shown to have a rich combinatorial structure. We are interested in a natural local Markov chain for sampling from the set of perfect matchings that is known to be ergodic and has been used in practice to discover properties of random fortresses. However, unlike related Markov chains used for sampling domino and lozenge tilings, this Markov chain on the square-octagon lattice appears to converge slowly. To understand why, we introduce a weighted version of the chain and prove that this chain can converge in polynomial time or exponential time depending on the settings of the parameters.

Keywords: Markov Chains, Sampling, Perfect Matchings, Probability, Fortress Model

1 Introduction

Perfect matchings arise in many natural computational contexts, and have been the cornerstone problem underlying many fundamental complexity questions. They are also of specific interest to the statistical physics community, where they are studied in the context of *dimer models*. Here, edges in a matching represent diatomic molecules, or dimers, and perfect matchings of a lattice region correspond to dimer packings. Physicists study the properties of these physical systems by relating fundamental thermodynamic quantities to weighted sums over the set of all configurations of the system, in our case the set of all perfect matchings of the lattice region.

The seminal work of Edmonds established that the decision and construction problems, i.e. efficiently deciding if a given graph has a perfect matching and finding it if so, were in P [6]. Subsequently, Valiant showed that counting perfect matchings is #P-complete, so it is believed that there is no such polynomial time general solution [26]. As a consequence, there has been a great deal of interest in finding both efficient *approximate* counting algorithms, as well as efficient exact counting algorithms in restricted settings. Jerrum, Sinclair and Vigoda showed how to approximately count and sample perfect matchings in any bipartite graph efficiently, although the complexity remains open on general graphs [13]. Alternatively, in 1969, Kasteleyn et al. developed a robust method to exactly count perfect matchings on any planar graph in polynomial time by calculating a Pfaffian on a directed version of the adjacency matrix [14, 25]. In fact, when the underlying graph is a lattice region, determinant-based methods for counting matchings have been shown to be even more efficient [8, 15].

Matchings on lattices arise naturally as well. For example, on finite regions of the hexagonal lattice, perfect matchings correspond to lozenge tilings of the dual region, and on finite regions of

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