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Expected reachability-time games

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ABSTRACT

Probabilistic timed automata are a suitable formalism to model systems with real-time, nondeterministic and probabilistic behaviour. We study two-player zero-sum games on such automata where the objective of the game is specified as the expected time to reach a target. The two players—called player Min and player Max—compete by proposing timed moves simultaneously and the move with a shorter delay is performed. The first player attempts to minimise the given objective while the second tries to maximise the objective. We observe that these games are not determined, and study decision problems related to computing the upper and lower values, showing that the problems are decidable and lie in the complexity class NEXPTIME ∩ co-NEXPTIME.

1. Introduction

Two-player zero-sum games on finite automata, as a mechanism for supervisory controller synthesis of discrete event systems, were introduced by Ramadge and Wonham [1]. In this setting the two players—called Min and Max—represent the *controller* and the *environment*, and controller synthesis corresponds to finding a winning (or optimal) strategy of the controller for some given performance objective. Timed automata [2] extend finite automata by providing a mechanism to model real-time behaviour, while priced timed automata are timed automata with (time-dependent) prices attached to the locations of the automata. If the game structure or objectives are dependent on time or price, e.g. when the objective corresponds to completing a given set of tasks within some deadline or within some cost, then games on timed automata are a well-established approach for controller synthesis, see e.g. [3–7].

In this paper we extend the above approach to a setting that is quantitative in terms of both timed and probabilistic behaviour. Probabilistic behaviour is important in modelling, e.g., faulty or unreliable components, the random coin flips of distributed communication and security protocols, and performance characteristics. We consider an extension of probabilistic time automata (PTA) [8–10], a model for real-time systems exhibiting nondeterministic and probabilistic behaviour.

In our model, called probabilistic timed game arena (PTGA), a token is placed on a configuration of a PTA and a play of the game corresponds to both players proposing a timed move of the PTA, i.e. a time delay and action under their control (we assume each action of the PTA is under the control of exactly one of the players). Once the players have made their choices, the timed move with the shorter delay is performed and the token is moved according to the probabilistic transition function of the PTA. Intuitively, players Min and Max represent two different forms of non-determinism, called *angelic* and

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demonic. To prevent the introduction of a third form, we assume the move of Max (the environment) is taken if the delays are equal. The converse can be used without changing the presented results.

Players Min and Max choose their moves in order to minimise and maximise, respectively, the objective function. The *upper value* of a game is the minimum expected time that Min can ensure, while the *lower value* of a game is the maximum expected value that Max can ensure. A game is *determined* if the lower and upper values are equal, and in this case the *optimal value* of the game exists and equals the upper and lower values.

The objectives frequently studied include reachability, which asks for certain locations to be eventually reached, safety, which asks for a given target set to be avoided, or more complex properties, expressed using a formula of a linear temporal logic. The objective function is then an indicator function saying whether the property is satisfied on a play, and the expected value then corresponds to the *probability* of the property being true. In our paper we are interested in a more complex setting and study *reachability-time* time objectives, which express the *expected time* to reach a given target set. These objectives have many practical applications, e.g., in job-shop scheduling, where machines can be faulty or have variable execution time, and both routing and task graph scheduling problems. For real-life examples relevant to our setting, see e.g. [7,11]. The reachability-time objectives are a special case of *weight* or *price* objectives in which different numbers are assigned to locations, and the value of the objective function depends on the respective numbers and the time spent in the locations; in our setting, the numbers are fixed to be 1 and the objective function simply sums the times spent in for each location. Computing properties related to price functions often leads to undecidability, even in non-probabilistic setting [12,13]. Studying simpler properties is thus motivated by the desire to obtain decidable properties while still being able to study sufficiently complex class of properties.

1.1. Contribution

We demonstrate the decidability of the problem of whether the upper (lower, or the optimal when it exists) value of a game with reachability-time objectives is at most a given bound. Our proofs immediately yield a NEXPTIME ∩ co-NEXPTIME complexity bound. To our best knowledge, this is the first decidability result for stochastic games on timed automata in which the objective concerns a random variable that takes non-binary values.

Our approach is based on extending the boundary region graph construction for timed automata [14] to PTGAs and demonstrating that the reachability-time problem can be reduced to the same problem on the boundary region graph. In particular, our proof aims to show that the limit of the step-bounded value functions in the timed automata and boundary region graph also coincide.

Generic results exist that allow one to prove that step-bounded values converge to the step-unbounded value, but to the best of our knowledge none are readily applicable in our setting where the state space is uncountable and little is known a priori about the value functions. For example, Banach fixpoint theorem requires the value iteration function (that takes a n-step value function and returns the $n+1$-step value function) to be a contraction on an underlying metric space, and it appears difficult to devise the metric space so that the contraction property is easily obtained. Another possible proof direction is Kleene fixpoint theorem, which requires Scott-continuity on the value functions, which again is a property that is difficult to establish in our setting. We are able to partly rely on the Knaster–Tarski fixpoint theorem which characterises the set of fixpoints, but it is not strong enough to prove the convergence itself, for reasons similar to the ones above. Several other theorems such as Brouwer fixpoint theorem or Kakutani fixpoint theorem are generally not suitable for proving properties that we require in turn-based stochastic games.

Hence, to prove that the limit of the step-bounded value functions is the desired value, we need to take a tailor-made approach. We first inductively show that, when the number of steps is bounded, then the value functions in timed automata and boundary region graph coincide and are non-expansive within a region. Here we make use of *quasi-simple functions* which generalise simple functions, previously used by Asarin and Maler in the study of games over non-probabilistic timed automata [3]. Then, using the non-expansiveness property, we show that the limit of the step-bounded value functions in the timed automata and boundary region graph also coincide. In this part we use Knaster–Tarski fixpoint theorem.

The definition of quasi-simple functions is a central component of our proof, as it is strong enough to enable us to utilise an approach used in proofs of fixpoint theorems, but on the other hand general enough to capture the values of reachability-time objectives. We believe that it can serve as a step from simple functions towards functions describing even more complex but still decidable objectives.

1.2. Related work

Hoffman and Wong-Toi [15] were the first to define and solve the optimal controller synthesis problem for timed automata. For a detailed introduction to the topic of qualitative games on timed automata, see e.g. [16]. Asarin and Maler [3] initiated the study of quantitative games on timed automata by providing a symbolic algorithm to solve reachability-time objectives. The works of [17] and [14] show that the decision problem for such games over timed automata with at least two clocks is EXPTIME-complete. The tool UPPAAL Tiga [6] is capable of solving reachability and safety objectives for games on timed automata. Jurdziński and Trivedi [18] show the EXPTIME-completeness for average-time games on automata with two or more clocks.