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Explore and repair graphs with black holes using mobile entities ☆,☆☆

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A B S T R A C T

In this paper, we study the problem of mobile entities that synchronously have to explore and repair a graph with faulty nodes, usually called *black-holes*, that destroy any entering entity. We consider the scenario where the destruction of an entity by means of a black-hole also affects all the entities within a fixed range r (in terms of number of edges), while the black-hole disappears. Clearly, if there are b black-holes in the graph, then $k \geq b$ entities are necessary to remove all of them from that graph. We ask for the minimum number of synchronous steps needed to make safe all the graph.

The results of this paper are both theoretical and experimental, and can be summarized as follows. From the theoretical point of view, first we show that the problem is NP-hard even for $b = k = 1$. Then, we provide a general lower bound holding when $r \geq 0$ and a higher one for the case of $r > 0$. We then consider the case of $r \leq 1$. We propose an optimal solution holding when k is unbounded, that is, an infinite number of robots is available. Then, we provide three different exploration strategies, named *snake*, *scout*, and *parallel-scout*, respectively, for the case of bounded k, that is, the number of robots is fixed a priori. The three strategies are then analyzed according to the time complexity with respect to the lower bound. From the experimental point of view, we implemented the three strategies and tested them on different scenarios with the aim of assessing their practical performance. The experiments confirm the theoretical analysis and show that *parallel-scout* is always by far the best exploration strategy in practice.

1. Introduction

The exploration task of unknown graphs by means of mobile entities has been widely considered during the last years. The increasing interest to the problem comes from the variety of applications that it meets. In robotics, it might be very useful to let a robot or a team of robots to explore dangerous or impervious zones. In networking, software agents might

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automatically discover nodes of a network and perform updates and/or refuse their connections. In this paper, we are interested in the exploration of a graph with faulty nodes, i.e. nodes that destroy any entering entity. Such nodes are called *black-holes*, and the exploration of a graph in such kind of networks is usually referred to as *black-hole search*. According to the assumed initial settings of the network, the knowledge, and the capabilities of the involved entities, many results have been provided in the literature.

1.1. Related works

Pure exploration strategies, without dealing with black-holes, have been widely addressed (see, for instance, [2–5] and references therein). In that case, the requirement is usually to perform the exploration as fast as possible. When black-holes are considered, along with the time (or equivalently the number of edge traversals) required for a full exploration, the main goal consists in minimizing the number of entities that may fall into the black-holes. A full exploration in this case means that, at the end, all the edges which do not lead to any black-hole must be "marked" as safe edges.

In this research area, many different models have been investigated. The system might be either synchronous [6,7] or asynchronous [8,9]. The input graph might be undirected [7,9] or directed [10,11]. It can be known in advance [7,12] or just a bound on the number of nodes is provided [11]. It can refer to a specific topology like rings [9,13], trees [6], hypercubes [14], tori [15]. Entities may communicate only when they meet [16], or by means of white-boards associated to the nodes of the graph [13], or simply by opportunely disposing available pebbles [17] or tokens [18]. The objective function may ask for the minimum number of entities, the minimum number of steps performed by all the entities, the minimum number of synchronous steps.

An interesting variation to the black-hole search problem has been recently introduced in [19], where the network must be decontaminated from *black-viruses*. A black-virus is a harmful process which destroys any agent arriving at the node where it resides; when that occurs, a black-virus moves, spreading to all the neighboring nodes, thus increasing its presence in the network. If however one of these nodes contains an entity, then that clone of the black-virus is destroyed. The initial location of the black-virus is unknown. The objective is to permanently remove any presence of the black-virus from the network with minimum number of node infections. The main cost measure is the total number of entities needed to solve the problem.

A hybrid model in between exploration and black-hole search has been introduced in [16], named the *Explore and Repair* problem. Given a graph G with n nodes, some of which are black holes, and a number k of entities, the idea is to perform the exploration of G as fast as possible with the constraint that if an entity enters in a black-hole, then it is destroyed and the black-hole disappears. The objective is to provide an exploration strategy that ensures to make safe the whole graph (i.e. removing all the black-holes) in the fastest way. A particular assumption is made in [16] on the consequences of entering in a black-hole. In fact, if more than one entity enter in a black-hole, only one gets destroyed while the others continue the exploration. With this assumption, a deterministic algorithm for the *Explore and Repair* problem is proposed in [16] which terminates within $O(\frac{n}{k} + \frac{\log f}{\log\log f} D)$ steps, where $f = \min\{\frac{n}{k}, \frac{n}{D}\}$, and D is the diameter of G. This algorithm is also shown to be worst-case asymptotically optimal, by giving a network such that for any deterministic algorithm, there is a placement of the faulty nodes forcing the algorithm to work for $\Omega(\frac{n}{k} + \frac{\log f}{\log\log f} D)$ steps. A general lower bound of $\Omega(\frac{n}{k} + D)$ steps is also provided.

It is possible to think about different scenarios where the model fits the assumption of [16], like in the case that software agents move along a network and, when they find a virus (represented by a black-hole), only one of them prolongs its stay for repairing purposes. However, it is also possible to think about scenarios where the assumption of [16] cannot model reality. For instance, in the case that entities are mobile robots exploring an impervious area disseminated of land-mines, if more than one robot incur concurrently in an explosion, then all of them get involved. Furthermore, the explosion may also affect robots within a fixed range.

1.2. Contribution of the paper

In this paper, we consider the Explore and Repair problem, and extend the model of [16] in two ways: (i) if more than one entity enters a black hole, then all of them are destroyed; (ii) given an integer number r, all entities within distance r (in terms of number of edges) from an entity entered in a black-hole instantaneously disappear from the network along with the black-hole. We call this the *Explore and Repair* problem with *radius* r. Clearly, if there are b black-holes, then $k \geq b$ entities are necessary to remove all the black-holes from the network. It might happen that the network is not explored completely, but an exploration algorithm must guarantee that all the black-holes have been removed as long as $k \geq b$.

The results of this paper are both theoretical and experimental, and can be summarized as follows. From the theoretical point of view, first we show that the problem is NP-hard even for $b = k = 1$; second, we consider the case of $r = 0$ and propose a simple variation of the strategy proposed in [16] that can be applied to give a worst-case asymptotically optimal solution when $k \geq 2b$; third, we consider the case of $r > 0$, and provide a lower bound of $\Omega(\frac{n}{k} + D + r\min\{D, b\} + |K_{\max}|)$ time steps, where b is the number of black holes, D is the diameter of the input graph, and $|K_{\max}|$ is the size of its maximum clique; fourth, we consider the case of $r \leq 1$ with an unbounded or a bounded number of entities k. In the unbounded case, that is, an infinite number of robots is available, we give an optimal strategy. For the bounded case,

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