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Partial pullback complement rewriting

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ABSTRACT

In this article, we first characterize the existence of certain pullback complements in a category, in terms of the existence of exponentials in a corresponding slice category and also in terms of the existence of pullbacks in the corresponding partial morphism category. We also prove that certain pullback complements exist in a weak adhesive VK category if and only if they exist in the corresponding partial morphism category.

We then propose a new graph rewriting technique, called the partial pullback complement rewriting, that uses pullback complements in a partial morphism category. We finally compare this with the double-pullback, single pushout, double-pushout and sesqui-pushout techniques, proving that in certain categories and under certain conditions, the partial pullback complement rewriting, implies and/or is implied by the above mentioned rewritings. Some illustrative examples are also given.

1. Introduction and preliminaries

In the field of graph transformation systems, there are several categorical approaches to graph rewriting. These are the pullback (PB), [2,3], double-pullback (DPB), [9], single-pushout (SPO), [7,15], double-pushout (DPO), [5,8], sesqui-pushout (SQPO), [4] rewritings; and rewriting in the span categories, [16], etc. All of these approaches are based on the categorical concept of pullback, pushout and/or pullback complement.

The goal of this article is to propose a new categorical approach to rewriting, called the partial pullback complement rewriting (PPBC), in such a way that generalizes all the above approaches at least in certain cases. This technique uses pullback complements in the partial morphism category of the background category and gives a one-square characterization of graph rewriting for right linear rules; thus unifying the main algebraic/categorical approaches to graph transformation.

It is known that in certain categories, the DPO, SPO and the SQPO rewritings coincide for left regular rules, [4]. Also SPO and DPO have been compared in [12]. We compare the PPBC approach with the DPB, SPO, DPO and SQPO rewritings and we prove that in certain categories and for certain rules and arbitrary matches, the object obtained by PPBC rewriting is a quotient object of the one obtained by SQPO, DPO or SPO rewritings. We also show that in certain categories and for certain rules and matches, under what conditions, the existence of PPBC implies and/or is implied by SQPO, DPO, DPB and SPO.

The PB rewriting and the rewriting in span categories are omitted in our comparison, as in the former case the comparison can be made via SQPO, see [4] and in the latter, since we are only dealing with whole matches, the situation reduces to the SQPO rewriting.

Through some examples we show that the results obtained for the categories under investigation, do not necessarily hold for more general categories. To this end, we recall:

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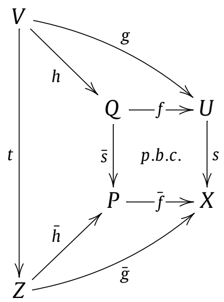
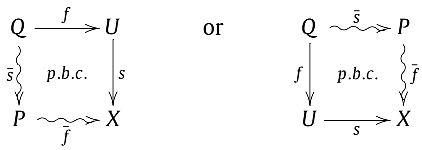
- In the following diagram, $(\bar{s}, \bar{f})$ is a pullback complement for (f, s), provided that the right square is a pullback, and if the outer square is a pullback and the upper triangle commutes, then there is a unique $\bar{h} : Z \to P$ rendering the left square and the lower triangle commutative, [6].



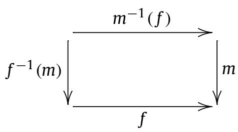
Then we denote such a pullback complement square as follows.



- For objects A and B in a category $\mathcal{C}$, we say the exponential B^A exists if the functors

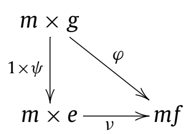
$$(\mathcal{C}_A)^{op} \underset{\mathcal{C}(-,B^A)}{\overset{\mathcal{C}(-\times A,B)}{\rightrightarrows}} Set$$

are naturally isomorphic, where $\mathcal{C}_A$ is the full subcategory of $\mathcal{C}$ with objects those objects of $\mathcal{C}$ whose product with A exist. If A is productive, i.e., its product with every object exists, then $\mathcal{C}_A = \mathcal{C}$, [17].
- A morphism $m : A \to B$ in a category $\mathcal{C}$, considered as an object in the slice category $\mathcal{C}/B$, is productive in $\mathcal{C}/B$ if and only if it is a universal morphism in $\mathcal{C}$, i.e., its pullback along every morphism in $\mathcal{C}$ exists, [17].
- A pullback of a map m along a map f is denoted by $f^{-1}(m)$. Since pullbacks are isomorphic, any two representatives, $f^{-1}(m)$, are isomorphic; and so throughout the article all the arguments are independent of this choice. For more on the inverse image operation, see [17].
- For objects $f : C \to B$ and $m : A \to B$ in the slice category $\mathcal{C}/B$, the product $f \times m$ in $\mathcal{C}/B$ is the diagonal of the following pullback square in $\mathcal{C}$.



i.e., $f \times m = ff^{-1}(m) = mm^{-1}(f)$, [17].

Lemma 1.1. *Given morphisms $f : C \to A$ and $m : A \to B$ in a category $\mathcal{C}$, $e = (mf)^m$ exists in $\mathcal{C}/B$ if and only if there exists a morphism $\nu : m \times e \to mf$ in $\mathcal{C}/B$, called the evaluation map, such that for any $g \in \mathcal{C}/B$ and for any morphism $\varphi : m \times g \to mf$ in $\mathcal{C}/B$, there exists a unique morphism $\psi : g \to e$ such that the following triangle commutes.*



Furthermore if m is mono, then the evaluation map is an isomorphism.

Proof. For the first part see [17]. For the second part, suppose m is mono. Then $m \times mf = mf$ and so $1_C : m \times mf \to mf$ is a morphism in $\mathcal{C}/B$. Therefore there exists a unique morphism $g : mf \to e$ such that the following triangle commutes.