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Bigraphs with sharing

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ABSTRACT

Bigraphical Reactive Systems (BRS) were designed by Milner as a universal formalism for modelling systems that evolve in time, locality, co-locality and connectivity. But the underlying model of location (the place graph) is a forest, which means there is no straightforward representation of locations that can overlap or intersect. This occurs in many domains, for example in wireless signalling, social interactions and audio communications. Here, we define *bigraphs with sharing*, which solves this problem by an extension of the basic formalism: we define the place graph as a directed acyclic graph, thus allowing a natural representation of overlapping or intersecting locations. We give a complete presentation of the theory of bigraphs with sharing, including a categorical semantics, algebraic properties, and several essential procedures for computation: bigraph with sharing matching, a SAT encoding of matching, and checking a fragment of the logic **BiLog**. We show that matching is an instance of the **NP**-complete sub-graph isomorphism problem and our approach based on a SAT encoding is also efficient for standard bigraphs. We give an overview of BigraphER (Bigraph Evaluator & Rewriting), an efficient implementation of bigraphs with sharing that provides manipulation, simulation and visualisation. The matching engine is based on the SAT encoding of the matching algorithm. Examples from the 802.11 CSMA/CA RTS/CTS protocol and a network management support system illustrate the applicability of the new theory.

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1. Introduction

Bigraphical Reactive Systems [1] were designed by Milner as a universal formalism for modelling interacting systems that evolve in time and space. They provide intuitive representations for systems with evolving locality, co-locality, and connectivity. In the course of working with practical applications of bigraphs for modelling different aspects of wireless networking protocols and management, we discovered there was no straightforward way to represent overlapping or intersecting spatial locations in bigraphs. But, overlaps are fundamental to wireless signalling and to other domains such as social interactions and audio communications. Consider for instance, the wireless network drawn in Fig. 1a. A spatial model based on trees cannot represent the fact that machine A is currently occupying a space location covered by both A's signal and B's signal, but not by C's signal.

We have therefore extended standard bigraphs to *bigraphs with sharing*, which allow for overlapping locations. This requires defining the place graphs of BRS as directed acyclic graphs (DAGs), instead of forests. An example place graph with sharing representing the topology of the wireless network in Fig. 1a is given in Fig. 1b. The three stations are denoted by A, B, and C, while S indicates wireless signal ranges. We will describe in detail this notation in the following section.

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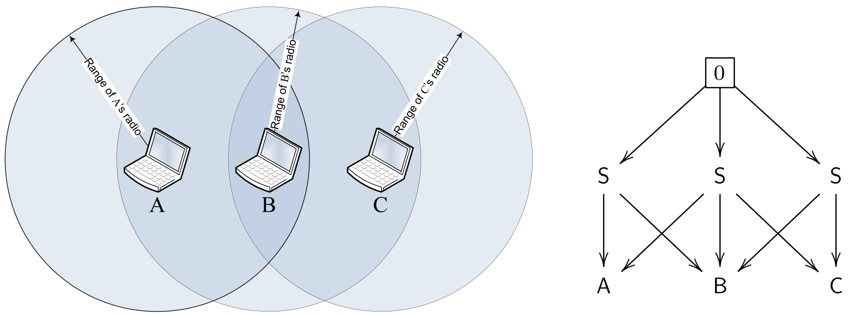
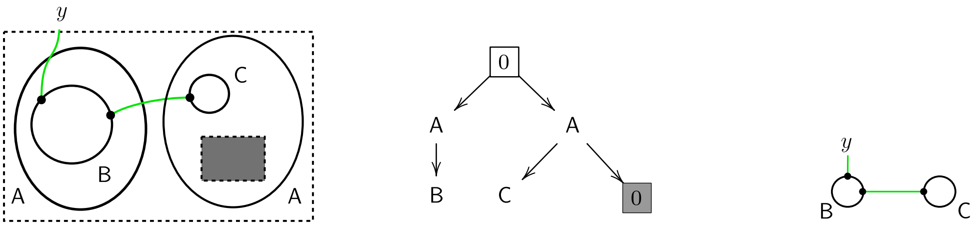


(a) Wireless network (b) Place graph with sharing

Fig. 1. Diagram of a wireless network of three stations (a) and a place graph with sharing representing its topology (b).



(a) $B : \langle 1, \emptyset\rangle \to \langle 1, \{y\}\rangle$ (b) $B^{\mathsf{P}} : 1 \to 1$ (c) $B^{\mathsf{L}} : \emptyset \to \{y\}$

Fig. 2. Example bigraph B (a) and its constituents: place graph B^{P} (b) and link graph B^{L} (c).

In this paper we give a complete presentation of the theory of bigraphs with sharing, including a categorical semantics, algebraic properties, and several essential procedures for computation: bigraph with sharing matching, a SAT encoding of matching, and checking a fragment of the logic **BiLog**. In each case we build on the existing results for (standard) bigraphs and our extensions are conservative in the sense that they still apply to standard bigraphs. We also give an overview of our implementation, called BigraphER, which provides an efficient implementation of computation, simulation, and visualisation for bigraphs with sharing.

When it is necessary to distinguish the different types of bigraphs, we refer to bigraphs as defined by Milner as *standard bigraphs*. We do not repeat the formal definitions here, but refer the reader to [1].

The paper is organised as follows. In the next Section we give an informal overview of bigraphs with sharing and discuss why an explicit representation of sharing is required. In Section 3 we show that the new composition and tensor product operators retain the same properties as the standard operators, and then we give a categorical semantics and results concerning algebraic forms and normalisation. In Section 4 we define a matching algorithm for bigraphs with sharing and give an overview of a SAT encoding. Our approach leads to a more efficient solution for standard bigraphs, compared with the standard approach that is based on inference. In Section 5 we show how checking a substantial fragment of the logic **BiLog** can be reduced to our matching, and in Section 6 we give an overview of BigraphER, our implementation of BRS with sharing. Finally, in Section 7 we discuss applications of bigraphs with sharing and in Sections 8 and 9 we discuss related work and our conclusions and plans for future work.

2. Bigraphs with sharing

2.1. Informal overview of standard bigraphs and bigraphs with sharing

The graphical form of an example standard bigraph B is drawn in Fig. 2a. Entities, real or virtual, are encoded by *nodes*, represented as ovals and circles. Their spatial placement is described by node nesting. Nodes are assigned a type, called *control*, denoted here by the labels A, B and C. A set of controls identifies a *signature*. Interactions between agents are represented by *links* like, for instance, the edge connecting the B-node and the C-node. Each node can have zero, one or many *ports*, indicated by bullets. Nodes of the same control have also the same number of ports. Dashed rectangles denote *regions*, also called *roots*. The rôle of a region is to specify adjacent parts of the system. Shaded squares are called *sites*. They encode parts of the system that have been abstracted away. Note that there is no significance in where a link crosses the boundary of a node in a bigraph. A bigraph can have *inner names* and *outer names*. In our example, y is an outer name. By convention in the graphical form, inner names and outer names are drawn below and above the bigraph, respectively. They encode links (or potential links) to other bigraphs representing the external environment or context. Elements forming a bigraph, namely nodes and edges, can be assigned unique identifiers, collectively called the *support* of a bigraph. When a bigraphical structure is assigned a support, it is called *concrete*.

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