Doctopic: Algorithms, automata, complexity and games ARTICLE IN PRESS TCS:9941

Theoretical Computer Science ••• (••••) •••–•••

Contents lists available at ScienceDirect

Theoretical Computer Science

www.elsevier.com/locate/tcs

On a class of covering problems with variable capacities in wireless networks

Selim Akl[a,2], Robert Benkoczi[b,*,1], Daya Ram Gaur[b], Hossam Hassanein[a,1], Shahadat Hossain[b,1], Mark Thom[b]

[a] *School of Computing, 557 Goodwin Hall, Queen's University, Kingston, Ontario, K7L 2N8, Canada*
[b] *Department of Mathematics and Computer Science, University of Lethbridge, 4401 University Dr, Lethbridge AB, T1K 3M4, Canada*

A R T I C L E I N F O

Article history:
Received 15 April 2014
Received in revised form 29 September 2014
Accepted 28 October 2014
Available online xxxx

Keywords:
Facility location
Capacitated covering
Approximation algorithms
Column generation
Complexity
Wireless networks

A B S T R A C T

We consider the problem of allocating clients to base stations in wireless networks. Two design decisions are the location of the base stations, and the power levels of the base stations. We model the interference, due to the increased power usage resulting in greater serving radius, as capacities that are non-increasing with respect to the covering radius. Clients have demands that are not necessarily uniform and the capacity of a facility limits the total demand that can be served by the facility. We consider three models. In the first model, the location of the base stations and the clients are fixed, and the problem is to determine the serving radius for each base station so as to serve a set of clients with maximum total profit subject to the capacity constraints of the base stations. In the second model, each client has an associated demand in addition to its profit. A fixed number of facilities have to be opened from a candidate set of locations. The goal is to serve clients so as to maximize the profit subject to the capacity constraints. In the third model, the location and the serving radius of the base stations are to be determined. There are costs associated with opening the base stations, and the goal is to open a set of base stations of minimum total cost so as to serve the entire demand subject to the capacity constraints at the base stations. We show that for the first model the problem is NP-complete even when there are only two choices for the serving radius, and the capacities are $1, 2$. For the second model, we give a $1/2$ approximation algorithm. For the third model, we give a column generation procedure for solving the standard linear programming model, and a randomized rounding procedure. We establish the efficacy of the column generation based rounding scheme on randomly generated instances.

1. Introduction

Given a set of client locations in some metric space, the covering facility location problem is to determine an optimal location for a set of facilities that are required to serve the clients. Facilities can serve clients within a prescribed radius

* Corresponding author.
E-mail addresses: akl@cs.queensu.ca (S. Akl), benkoczi@cs.uleth.ca (R. Benkoczi), gaur@cs.uleth.ca (D.R. Gaur), hossam@cs.queensu.ca (H. Hassanein), hossain@cs.uleth.ca (S. Hossain), thom@cs.uleth.ca (M. Thom).

1 The author acknowledges the funding support received for this research from NSERC.
2 The author acknowledges the funding support received for this research from Queen's University.

http://dx.doi.org/10.1016/j.tcs.2014.10.044

only, *i.e.* a client is *covered* or served by a single facility if it is within the covering range of the facility, otherwise it is not covered. The covering problem was introduced in [4] and it has been widely used in practice in areas such as the location of emergency vehicles, of retail facilities, or of telecommunication equipment. However, the simple "all or nothing" covering constraint has been found too restrictive for many applications, and several relaxations have been proposed and studied in the last decade. The excellent survey [2] presents three relaxations: (a) the *gradual cover* model where the degree with which a client is served decreases as its distance to the facility increases; (b) the *cooperative cover* model where several facilities can contribute to serving the same client; (c) the *variable covering radius* model where the planner can choose the covering range for the facilities, but the opening cost for the facility increases with its range.

We introduce a new family of covering problems, Covering with Variable Capacities (*CVC*), which addresses the client coverage problem in the presence of interference in wireless networks. *CVC* generalizes the classical capacitated covering due to [15] where an upper bound on the total demand that can be served by a facility is imposed. Facilities correspond to wireless base stations employing omni-directional antennas and clients represent service subscribers. We assume that the location of the clients is given. Demands (bandwidth requirements) and profits (revenue) are associated with the clients.

In our model, every facility has a variable covering range and the facilities need to be located and assigned a covering range. The range can only be increased at the expense of the capacity, as increasing the power of the radio transmitter causes more interference in the network [3,9,10].

Our paper is motivated by the current research in the field of networking aimed at understanding the connection between interference and base station capacity in networks utilizing the popular code division multiple access (CDMA) technology. Several researchers have investigated the idea of using this dependency to improve the performance of networks. For example, Radwan and Hassanein [16] show that there are significant savings in resource utilization for wide band CDMA networks when the range of the base stations is appropriately chosen, and Tam et al. [18] describe a cellular network that exploits this phenomenon. Arguably our models and solutions have immediate applications in the mobile telephony industry. In addition we extend the theory of facility location problems by studying a new class of location problems in which the facilities have variable capacities, and the designer not only has to choose a location for the facility but also the capacity at which the facility should operate.

Problem *CVC* can be defined in any metric space. We study three variants of *CVC*: *CVC* with fixed facilities (or simply *CVC*) where the location of the facilities is given and the objective is to maximize the total profit of the clients served, maximum *CVC* where a set of clients with maximum total profit must be covered by a fixed number of facilities, and set-cover *CVC* where the entire set of clients must be covered by a set of facilities with total minimum cost. The three problems are defined next. We use index notation to refer to clients and facilities, and so u_i for some index i refers to a client and a_j for some index j refers to a facility.

Problem 1 *(*CVC*).*
Input:

- A set $\mathcal{U} = \{u_i : i \in \mathcal{I}\}$ of clients where $\mathcal{I}$ is the index set of clients.
- For each client u_i, a non-negative demand or size s_i and profit p_i.
- A set $\mathcal{A} = \{a_j : j \in \mathcal{J}\}$ of open facilities, where $\mathcal{J}$ is the index set of facilities.
- For each facility a_j, a set R_j of allowed ranges and for each $r \in R_j$ a corresponding capacity c_{jr}. We denote by N_{jr} the set of clients within the covering range r of facility a_j.

Output:

- For each facility a_j, a range $r_j \in R_j$.
- For each facility a_j, a subset of clients denoted by $\mathcal{I}_j \subseteq \mathcal{I}$ that are served exclusively by a_j satisfying $\sum_{i \in \mathcal{I}_j} s_i \le c_{jr_j}$ and $u_i \in N_{jr_j}$ for all $i \in \mathcal{I}_j$.

Objective:

- To maximize the total profit of clients served, $\max \sum_{i \in \bigcup_{j \in \mathcal{J}} \mathcal{I}_j} p_i$.

Problem 2 *(Maximum* CVC*).*
Input:

- Same as for Problem 1. The set of facilities represents *candidate* facility locations.
- A positive integer k.

Output:

- k facilities are to be opened, indexed by $\mathcal{J}^* \subseteq \mathcal{J}$ where $|\mathcal{J}^*| = k$.