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Parametric metric interval temporal logic ☆

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ABSTRACT

In this paper we focus on the role of parametric constants in real-time temporal logic and introduce the logic PMITL as a parametric extension of MITL. For this logic, we study decision problems which are the analogues of satisfiability, validity and model-checking problems for non-parametric temporal logic. We impose some restrictions on the use of the parameters: each parameter is used with a fixed polarity, parameters can appear only in one of the endpoints of the intervals, parametric linear expressions can be used only as right endpoints of the intervals. We show that, for parameter valuations yielding only non-singular intervals, the considered problems are all decidable and Expspace-complete, such as for the decision problems in MITL. Moreover, we show that if we relax any of the imposed restrictions, the problems become undecidable. We also investigate the computational complexity of these problems for natural fragments of PMITL, and show that for some meaningful fragments they can be solved in polynomial space and are Pspace-complete. Finally, we consider the decision problem of determining the truth of first-order queries over PMITL formulas where the parameters are used as variables that can be existentially or universally quantified. We solve this problem in several cases and exhibit an exponential-space algorithm.

1. Introduction

Temporal logic is a simple and standard formalism to specify the desired behavior of a reactive system. Its use as a specification language was first suggested by Pnueli [24] who proposed the propositional linear temporal logic (LTL). This logic presents natural operators to express temporal requests on the time ordering of occurrences of events, such as "always", "eventually", "until", and "next".

The logics MTL [20] and MITL [3] extend LTL with a real-time semantics where the changes of truth values happen according to a splitting of the line of non-negative reals into intervals. Syntactically, these logics augment the temporal operators of LTL (except for the next operator which has no clear meaning in a real-time semantics) with a subscript which expresses an interval of interest for the expressed property. Thus, properties such as "every time an a occurs then a b must occur within time $t \in [3, 5]$" become expressible. To gain the decidability, the singular intervals, or equivalently the equality, are not allowed as subscripts of the temporal operators in MITL [3].

In this paper, we extend MITL with parametric constants, i.e., we allow the intervals in the subscripts of the temporal operators to have as an endpoint a *parametric expression* of the form $c + x$, for a parameter x and a constant c. Therefore,

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typical time properties which are expressible in MITL can now be analyzed by varying the scope of the temporal operators depending on the values of the parameters. As an example, consider a parameterized version of the above property: "every time an a occurs then a b must occur within a time $t \in [3, 5 + x]$" where x is a parameter. One could be interested in determining if there exists a value of x such that this property holds on a given timed sequence, or if this is true for any possible value of x, or more, the set of x values such that the property holds.

We define a logic, which we denote PMITL (parametric MITL), where in each interval at most one endpoint can be a parametric expression, and such that each parameter is used with a fixed "polarity". The concept of polarity is semantic and is related to whether the space of the values for a parameter such that the formula is satisfied is upward or downward closed. For example, the set of values of x for the assertion "an a will eventually occur within time x" is upward closed: a model which satisfies this for $x = 3$ also satisfies it for every $x > 3$.

For the introduced logic, we study the decision problems which are the analogues of satisfiability, validity and model-checking for non-parametric temporal logic. As models for our formulas, we consider the class of timed automata augmented with parameters, which are called L/U automata [17] and share the same kind of restrictions on the parameters as PMITL. Note that in the model-checking problem we allow parameters both in the formula and the model.

We define our decision problems as the universality and the emptiness problems for classes of sets of parameter valuations. In particular, we define the set of all the admissible valuations such that a formula is satisfiable and the set of all the admissible valuations such that an L/U automaton is a model of the formula (in the following, referred to as S-sets), and the analogous sets which guarantee instead the validity of a given formula, possibly with respect to a given L/U automaton (in the following, referred to as V-sets). Here, a parameter valuation is admissible if it evaluates a parameterized interval to neither an empty nor a singular set. As in MITL, the restriction to non-singular intervals is required for the decidability of the decision problems. The other restriction is not critical for decidability, and has the only purpose to rule out some degenerate cases: an empty interval would make the related assertion equivalent to either the constant true or false.

Our main result is that the universality and the emptiness problems for the S-sets and the V-sets are decidable and Expspace-complete. The result still holds if we allow linear expressions of the parameters, of the form $c_1x_1 + \ldots + c_nx_n$, provided that such expressions are used only as right endpoints of the parameterized intervals. The proof goes through a reduction to decidable problems for Büchi L/U automata [7]. Recall that satisfiability, validity and model-checking problems for MITL formulas [3] are Expspace-hard, thus this result shows that adding parameters to MITL, though augmenting the expressiveness, does not affect the abstract computational complexity of its decision problems.

One of the crucial points concerning parametric systems is to explicitly compute the set of all the valuations that make a parameterized formula satisfiable (or valid). In [2] this problem has been solved for a parametric logic (PLTL) with a discrete semantics when all the operators have the same polarity, using an algorithm that takes double-exponential time in the number of parameters. It is very interesting to seek algorithms which get more information on this set of parameter valuations also when parameters of both polarities are allowed. We make a progress in this direction by studying a general decision problem over the S-sets and V-sets (that includes the above emptiness and universality problems as particular cases).

For a given formula with parameters $x_1, \ldots, x_n$, we study queries of the form $Q_1x_1 \ldots Q_nx_n$. over the corresponding S-sets and V-sets, where each Q_i is either an existential or a universal quantifier. For each i, the quantifier Q_i bounds, either existentially or universally, the values that a parameter x_i may be assigned within the given set. The already discussed non-emptiness problem corresponds to queries with only existential quantifiers, and the universality problem corresponds to queries with only universal quantifiers. We prove that also this generalized decision problem is not harder than the basic problems and that this holds not only when all the parameters have the same polarity but also in several more general cases.

We refine our complexity results by addressing the complexity of some fragments of PMITL. On the positive side, we prove that some of the considered problems are in Pspace for two expressive fragments. In particular, we prove that the following problems are in Pspace: (1) deciding the emptiness of S-sets for formulas where the only parameterized operators are either of the form $\Diamond_{(c,d+x)}$, or one of the endpoints of its interval is 0 or ∞, (2) deciding the universality of V-sets in the fragment where the only parameterized operators are either of the form $\Box_{(c,d+y)}$, or one of the interval endpoints is 0 or ∞. These fragments are quite expressive (for example we can express properties such as the parameterized response property seen above) and contain the logic considered in [7]. To the best of our knowledge, the union of these fragments captures the most general known formulation of parametric constraints in PMITL with the considered decision problems in Pspace.

We complete our analysis on the complexity of the fragments by showing that the considered decision problems are Expspace-hard for the fragment $\text{PMITL}_{0,\infty}$ where each non-parametric interval has one endpoint which is either 0 or ∞, and that hardness still holds in the fragments of $\text{PMITL}_{0,\infty}$ with only parametric operators of one polarity.

In the definition of PMITL we impose some restrictions on the parameters. In particular, we do not allow a parameter valuation to evaluate a parametric interval to a singular interval, each parameter is used with a fixed polarity consistently through all the formula, and in each interval we can use parameters in only one of the endpoints. We show that if we relax any of these restrictions the resulting problems become undecidable. In fact, it is known that testing parameters for equality (and thus allowing singular intervals) leads to undecidability (see [2]). Also, if a parameter is used with both polarities in a formula, then it is possible to express equality, and again the problems become undecidable. We thus consider some natural ways of defining parameterized intervals with parameters in both the endpoints (i.e., both the endpoints are added with