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“Robot Cloud” gradient climbing with point measurements ☆

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ABSTRACT

A scalar-field gradient climbing process for a large group of simple, low-capability mobile robotic agents using only point measurements is proposed and analyzed. The agents are assumed to be memoryless and to lack direct communication abilities. Their only implicit form of communication is by sensing the position of the members of the group. The proposed gradient following algorithm is based on a basic gathering algorithm. The gathering algorithm is augmented by controlling the agents' speed as follows: agents that sense a higher value of the field move slower toward the group center than those sensing lower values, thereby causing the swarm to drift in the direction of the underlying field gradient. Furthermore, a random motion component is added to each agent in order to prevent gathering and allow sampling of the scalar field. We prove that in the proposed algorithm, the group is cohesive and indeed follows the gradient of the scalar field. We also discuss an algorithm based on a more restrictive sensing capabilities for the agents.

1. Introduction

In this work we consider the problem of designing the control for a group of robots so that they will follow the gradient of a scalar field defined over d-dimensional space $\rho : \mathbb{R}^d \to \mathbb{R}$ where $d \in \mathbb{N}$. Our claim is to have the group motion as the result of simple memoryless interactions between the robotic agents. It is often assumed that the scalar field is the concentration of some chemical material and is generated by a diffusion process originating at a source. Therefore, the value of the scalar field becomes weaker as one moves away from it. Hence, the problem is sometimes called *source-seeking* with applications varying from tracking a plume of gas to finding the source of leakage of an hazardous chemical.

A large group of miniature (possibly nano) and very simple robotic agents with very low capabilities is considered. Very small robots, even equipped with multiple sensors, are unable to directly sense the gradient of $\rho(\cdot)$, the scalar field, because the spatial separation between the on board sensors is not large enough. Hence, it is assumed that the agents can only take point measurements of $\rho(\cdot)$. We limit the discussion to *memoryless*, or *reactive*, algorithms in the sense that an action performed by an agent is determined solely by the agent's sensors readings at the time the action is taken. Since they are *memoryless*, the agents cannot estimate the gradient by comparing the currently measured value to previously sensed values and, by assumption, they cannot communicate directly thus cannot explicitly share their point measurements.

Full mutual sensing of position is assumed, i.e. every agent can sense the relative position of all other agents in the group. This assumption might seem quite strange and restrictive, however our results prove that by tuning the algorithm parameters the swarm can be made as cohesive as desired. Specifically, the swarm can be made cohesive enough to guarantee that at (almost) all times, all agent pairs are close enough to allow mutual sensing of relative position (see the discussion in Section 5). Nevertheless, some alternatives to full position sensing are explored in Section 5.1.

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Alternatively, as formalized in Section 4, the proposed process can be described as a gathering or swarming algorithm augmented to allow gradient climbing. The agents follow the swarming algorithm while their speed is controlled by the scalar field, or in other words, the medium they travel in. Many swarming algorithms, including those used in this work, can be employed by considering potential fields which are both induced by the agents and dictate the agents movement. Hence, instead of sensing all other agents relative positions, each agent can simply react to the potential field induced by the other agents while its speed is controlled by the local scalar field.

2. Previous work

In a recent paper we have proposed a gradient climbing algorithm for two memoryless robotic agents using only implicit communication and point measurements [1]. The two agents considered were assumed to orbit their center of mass while the agent who senses the higher ρ-value strives to maintain a slightly larger distance from the center of mass. As a result, the robot pair was shown to drift toward higher ρ-values. The algorithm proposed here may be viewed as an extension of the same basic mechanism to the multi-agent scenario.

A broad survey of previous work on gradient following by multiple agents can be found in our previous paper [1]. Hence only the most relevant work will be discussed here. With a single agent, the common method used to overcome the point measurement limitation is by taking spatially separated measurements by moving the agent between readings. By "remembering" and subsequently comparing the readings, the agent can estimate the gradient, see e.g. [2,3]. As the agent moves, the field value it measures changes. Assuming continuous measurement, the time differential of the readings ($\dot{\rho}(\cdot)$) can be recorded and used to control the agent, see e.g. [4–6]. All the above under the assumption that the agent has memory.

In a multi-agent scenario where every agent is able to measure and communicate $\rho(\cdot)$ or the gradient of $\rho(\cdot)$, variants of the Artificial Potential Field framework can be employed [7,8]. Gazi and Passino[9] studied the behavior of a swarm of agents affected by attraction, repulsion and gradient climbing forces. They proposed rules of motion ensuring that the swarm maintains cohesiveness and travels in the direction of the gradient.

An algorithm that uses point measurements only was proposed in [10]. There, N agents maintain a uniform circle formation. By comparing the values they measure, the agents in the formation move with the gradient. However, in contrast to our work, in [10] every agent is assumed to have access to the values measured by all other agents. Recently, Berdhal et al. [11] observed gradient climbing behavior in schools of fish i.e. individual fish reduce their speed when sensing low light levels resulting in movement of the school toward darker regions of the aquarium. Inspired from this work, Wu et al. formalized the algorithm of [11] and proved convergence to a peak in the scalar field. Similarly to our work, Wu et al. [12] also assume full visibility and a linear scalar field, however, their algorithm is different from ours. Recently, Wu and Zhang shown that a switching strategy i.e. the agents switch between individual exploration and cooperative exploration improve the convergence time [13].

3. Preliminaries

The environment considered in this work is the d-dimensional space $\mathbb{R}^d$. For real scenarios it is natural to consider $1 \le d \le 3$. By assumption, there is a scalar field $\rho(\cdot)$ defined over the space $\rho : \mathbb{R}^d \to \mathbb{R}$. The agents' task is to follow the gradient of $\rho(\cdot)$. The system comprises $M \ge 2$ agents denoted by $r_1..r_M$. The location of agent r_i in a fixed global coordinate frame is given by X_i where

$$X_i = \left[x_i^1, x_i^2 ... x_i^d\right]^T \tag{1}$$

The unit vectors of the global coordinate system are denoted by $\hat{x}^1 ... \hat{x}^d$. Note that the global coordinate frame is unknown to the agents and is defined here solely for convenience of analysis. Let X_{cm} be the agents' center of mass, which, in the global coordinate system, is given by

$$X_{cm} = \frac{1}{M} \sum_{i=1}^{M} X_i \tag{2}$$

Let $\rho(X)$ denote the value of the scalar field at point X.

In our model time is discrete, i.e. $t = 0, 1, 2....$ It is assumed that the algorithm was initialized at time $t = 0$. When we explicitly add t to the indices of a quantity we refer to the value of that quantity at time t, e.g. $X_i(t)$ is the location of agent r_i at time t.

4. Gradient-climbing process

Consider the following function, the sum of all squared inter-robot distances

$$\Psi\{X_1, X_2 ... X_M\} \triangleq \frac{1}{2} \sum_{i,j} d(X_i, X_j)^2 \tag{3}$$