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The quantitative linear-time–branching-time spectrum

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ABSTRACT

We present a distance-agnostic approach to quantitative verification. Taking as input an unspecified distance on system traces, or executions, we develop a game-based framework which allows us to define a spectrum of different interesting system distances corresponding to the given trace distance. Thus we extend the classic linear-time–branching-time spectrum to a quantitative setting, parametrized by trace distance. We also prove a general transfer principle which allows us to transfer counterexamples from the qualitative to the quantitative setting, showing that all system distances are mutually topologically inequivalent.

1. Introduction

For rigorous design and verification of embedded systems, both qualitative and quantitative information and constraints have to be taken into account [23,26,30]. This applies to the *models* considered, to the *properties* one wishes to be satisfied, and to the *verification* itself. Hence the question asked in quantitative verification is not "Does the system satisfy the requirements?", but rather "*To which extent* does the system satisfy the requirements?" Standard qualitative verification techniques are inherently *fragile*: either the requirements are satisfied, or they are not, regardless of how close the actual system might come to the specification. To overcome this lack of robustness, notions of *distance* between systems are essential.

As pointed out in [23], qualitative and quantitative aspects of verification are best treated orthogonally in any theory of quantitative verification. In practical applications these aspects may indeed interfere with each other, but for the purpose of theory, they are best treated separately. The formalism we propose in this paper addresses this separation by modeling qualitative aspects using standard *labeled transition systems* and expressing the quantitative aspects using *trace distances*, or distances on system executions. Based on these ingredients, we develop a comprehensive theory of *system distances* which generalizes the standard linear-time–branching-time spectrum [17,18,36] to a quantitative setting, see Fig. 1. Similarly to [4], our theory relies on Ehrenfeucht–Fraïssé games and allows for a more refined analysis of systems. More precisely, our parametrized framework forms a hierarchy of games, for each trace distance used in its instantiation. In the quantitative setting, using games with quantitative objectives as opposed to discrete games, effectively allows us to obtain a continuous verdict on the relationship between systems, and hence to detect the difference between minor and major discrepancies between systems. We refer to [15] for a good introduction to the theory of quantitative games.

Indeed the view of this paper is that in a theory of quantitative verification, the quantitative aspects should be treated just as much as an input to a verification problem as the qualitative aspects are. Hence it is of limited use to develop a theory pertaining only to some *specific* quantitative measures like the ones in [2,3,11,24,33] and other papers which all treat only a few specific ways of measuring distances; any theory of quantitative verification should work just as well regardless

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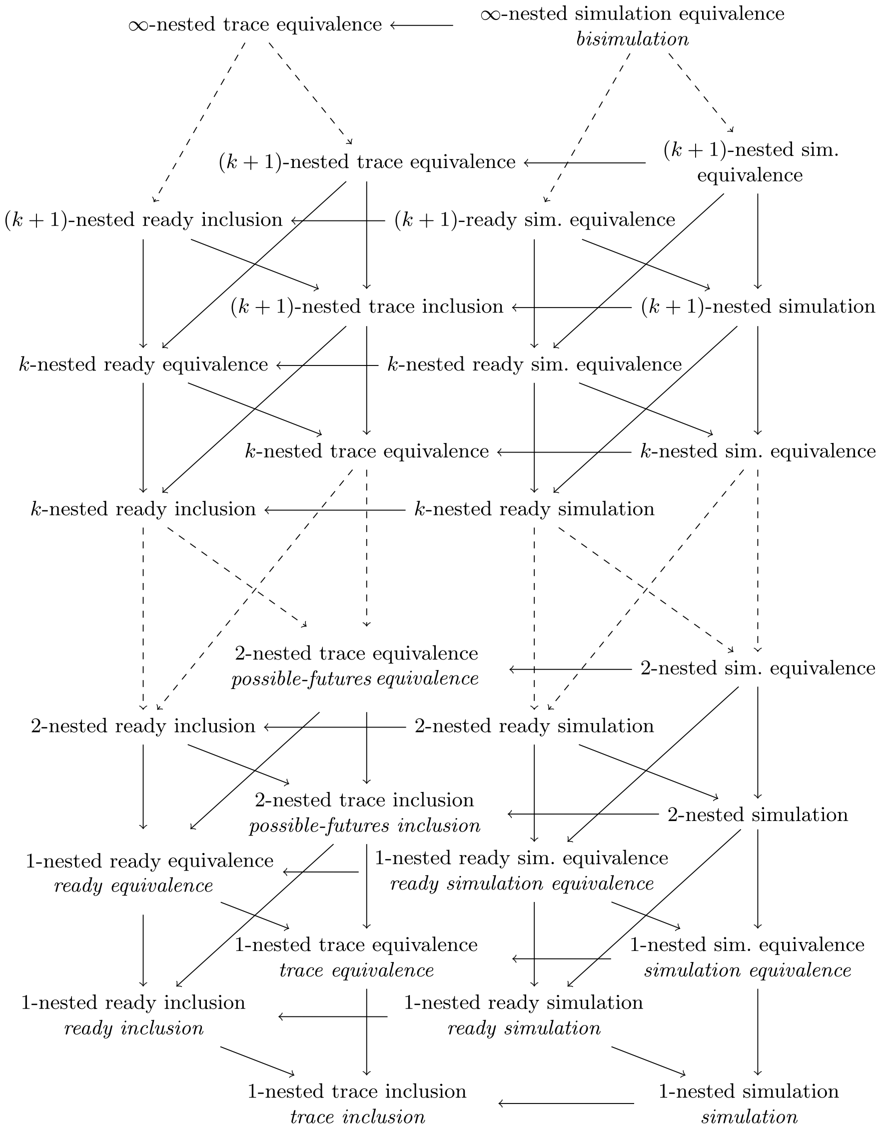

Fig. 1. The quantitative linear-time–branching-time spectrum. The nodes are the different system distances introduced in this paper, and an edge $d_1 \longrightarrow d_2$ or $d_1 \dashrightarrow d_2$ indicates that $d_1(s,t) \geqslant d_2(s,t)$ for all states s, t, and that d_1 and d_2 in general are topologically inequivalent.

of the way the engineers decide to measure differences between system executions. We note that the framework we lay out here may be equally instantiated with labels (or propositions) in states rather than on transitions, hence it also generalizes the formalisms of [5,35].

We take as input a labeled transition system and a trace distance; both are unspecified except for some general characteristic properties. Based on this information and using the theory of *quantitative games*, we lift most of the linear-time–branching-time spectrum of van Glabbeek [36] to the quantitative setting, while the rest may be obtained in a similar way using minor additional conditions as described in [4]. We show that all the distinct equivalences in van Glabbeek's spectrum correspond to topologically inequivalent distances in the quantitative setting.

As our framework is independent of the chosen trace distance, we are essentially adding a second, quantitative, dimension to the linear-time–branching-time spectrum. In this terminology, the first dimension is the qualitative one which concerns the different linear and branching ways of specifying qualitative constraints, and the second dimension bridges the gap between the trivial van Glabbeek spectrum in which everything is equivalent, and the discrete spectrum in which everything is fragile.

We start the paper by recalling some preliminaries and fixing notation in Section 2. Section 3 then shows that our general framework of trace distances is applicable to a large number of system distances found in the literature [2,3,5–8,10–12,20,22,24,32,33,35,37]; indeed we show in Section 8 that it generalizes all of them.

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