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Price of asynchrony in mobile agents computing

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ABSTRACT

Asynchrony is one of the main challenges in distributed computing. Some tasks, such as distributed Byzantine consensus, are impossible in the asynchronous setting, while they can be carried out synchronously. For other tasks, such as rendezvous in arbitrary graphs, the best known synchronous algorithm has cost much lower than the best asynchronous one. Various degrees of asynchrony and synchrony and comparisons between them in terms of feasibility of distributed tasks have been particularly intensely studied in the context of mobile agents computing. However, somewhat surprisingly, there are no results showing a provable difference of cost between the synchronous and asynchronous versions of a task executed by mobile agents.

The aim of this paper is to fill up this gap. We show for the first time that for some natural task executed by mobile agents in a network, the optimal cost of its deterministic solution in the asynchronous setting has higher order of magnitude than that in the synchronous scenario. The task for which we show this difference is well-studied: that of rendezvous of two agents in an infinite oriented grid. More precisely, we consider two agents with distinct integer labels starting at a distance D in the infinite oriented grid. Each agent knows D and its own label but not the label of the other agent and it does not know the position of the other agent relative to its own. Agents do not have any global system of coordinates. They have to meet in a node or inside an edge of the grid, and the cost of a rendezvous algorithm is the number of edge traversals by both agents until the meeting. We show that in the synchronous scenario rendezvous can be performed at cost $O(D\ell)$, where ℓ is the length of the (binary representation of the) smaller label, while cost $\Omega(D^2 + D\ell)$ is needed for asynchronous completion of rendezvous. Hence, for instances with $\ell = o(D)$, the optimal cost of asynchronous rendezvous is asymptotically larger than that of synchronous rendezvous.

1. Introduction

The background. Asynchrony is one of the main challenges in distributed computing. Some tasks, such as distributed Byzantine consensus, are impossible in the asynchronous setting [24], while they can be carried out synchronously. For other tasks, such as rendezvous in arbitrary graphs, the best known synchronous algorithm [36] has cost much lower than the best asynchronous one [23]. Various degrees of asynchrony and synchrony and comparisons between them in terms of feasibility of distributed tasks have been particularly intensely studied in the context of mobile agents computing. The investigated tasks range from pattern formation by mobile agents (sometimes called robots) [35,37] to collective graph exploration [26,31,34]

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and rendezvous or gathering [9,10,12,15,19,25,28]. However, somewhat surprisingly, there are no results showing a provable difference of cost between the synchronous and asynchronous versions of a task executed by mobile agents.

The aim of this paper is to fill up this gap. In order to do this, we consider the well-studied task of rendezvous of two agents in an infinite oriented grid. Two agents with distinct positive integer labels start at a distance D in the infinite oriented grid. Each agent knows D and its own label but not the label of the other agent and it does not know the position of the other agent relative to its own. Agents do not have any global system of coordinates. They have to meet in a node or inside an edge of the grid, and the cost of a rendezvous algorithm is the number of edge traversals by both agents until the meeting. In the synchronous scenario agents move in synchronous rounds, while in the asynchronous scenario an adversary controls the speed of each agent, can slow it down, speed it up, or hold it idle for an arbitrary finite time.

Our results. We give a positive answer to the following question:

> Do there exist natural tasks executed by mobile agents in networks, which can be completed both in the synchronous and in the asynchronous scenarios, but for which the cost in the asynchronous scenario is provably larger than in the synchronous scenario?

We show that for a well-studied task executed by mobile agents in a network, the optimal cost of its deterministic solution in the asynchronous setting has higher order of magnitude than that in the synchronous scenario. More precisely, we show that in the synchronous scenario rendezvous in an infinite oriented grid can be performed at cost $O(D\ell)$, where ℓ is the length of the (binary representation of the) smaller label, while cost $\Omega(D^2 + D\ell)$ is needed for asynchronous completion of rendezvous. Hence, for instances with $\ell = o(D)$, the optimal cost of asynchronous rendezvous is asymptotically larger than that of synchronous rendezvous. To the best of our knowledge, this is the first time when a task executed by mobile agents is shown to cost strictly more in the asynchronous than in the synchronous scenario.

Related work. In many papers on rendezvous the synchronous scenario was assumed, in which agents navigate in the graph in synchronous rounds. An extensive survey of randomized rendezvous in various scenarios can be found in [4], cf. also [2,3,5,6]. Deterministic rendezvous in networks has been surveyed in [33]. Several authors considered the geometric scenario (rendezvous in an interval of the real line, see, e.g., [11], or in the plane, see, e.g., [7,8]). Rendezvous of more than two agents, often called gathering, has been studied, e.g., in [20,22,32,38]. In [20] agents were anonymous, while in [38] the authors considered gathering many agents with unique labels. Gathering many labeled agents in the presence of Byzantine agents was studied in [22]. The problem was also studied in the context of multiple robot systems, cf. [13,25], and fault tolerant gathering of robots in the plane was studied, e.g., in [1,14]. Comparison of different degrees of asynchrony in the context of feasibility of gathering oblivious robots in the plane was discussed in [28].

For the deterministic setting a lot of effort has been dedicated to the study of the feasibility of rendezvous, and to the time required to achieve this task, when feasible. For instance, deterministic rendezvous with agents equipped with tokens used to mark nodes was considered, e.g., in [30]. Deterministic rendezvous of two agents that cannot mark nodes but have unique labels was discussed in [19,29,36]. These papers are concerned with the time of synchronous rendezvous in arbitrary finite connected graphs. In [19] the authors show a rendezvous algorithm polynomial in the size of the graph, in the length of the shorter label and in the delay between the starting times of the agents. In [29,36] rendezvous time is polynomial in the first two of these parameters and independent of the delay. In all cases the polynomials are quite large.

Asynchronous rendezvous of two agents in a network has been studied in [10,15,16,18,23,27]. In [18] the authors investigated the cost of rendezvous in the infinite line and in the ring. They also proposed a rendezvous algorithm for an arbitrary graph with a known upper bound on the size of the graph. This assumption was subsequently removed in [16], but both in [18] and in [16] the cost of rendezvous was exponential in the size of the graph. In [27] asynchronous rendezvous was studied for anonymous agents and the cost was again exponential. The first rendezvous algorithm working for arbitrary finite connected graphs at cost polynomial in the size of the graph and in the length of the shorter label was presented recently in [23]. In [10,15] the authors restricted attention to infinite oriented multidimensional grids and used the assumption that each agent knows its starting position in a global system of coordinates. Under this powerful hypothesis they constructed asynchronous rendezvous algorithms working at cost polynomial in the initial distance.

A different asynchronous scenario was studied in [12,25] for the plane. In these papers the authors assumed that agents are memoryless, but they can observe the environment and make navigation decisions based on these observations. A similar scenario for the ring was used in [17].

2. The synchronous and asynchronous models

In this section we define precisely the synchronous and the asynchronous models for rendezvous in networks, for which we will show the gap between costs. Both these models have been extensively used in the literature, the synchronous model, e.g., in [19,29,36] and the asynchronous model, e.g., in [10,15,16,18,23].

We start with the underlying graph used in this paper, which is the same for both models. This is the infinite oriented orthogonal grid defined by the following embedding in the Euclidean plane. Every node u of the grid is adjacent to 4 nodes at Euclidean distance 1 from it, and located North, East, South and West from node u. Ports at every node are coherently labeled N, E, S and W. Nodes are unlabeled and agents cannot mark visited nodes in any way. Hence agents do not learn