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Optimal wavelength assignment in the implementation of parallel algorithms with ternary n-cube communication pattern on mesh optical network

Cui Yu [a], Xiaofan Yang [a,b,*], Li He [a,c], Jing Zhang [a]

[a] College of Computer Science, Chongqing University, Chongqing, 400044, China
[b] School of Electronic and Information Engineering, Southwest University, Chongqing, 400715, China
[c] College of Computer Science, Chongqing University of Posts and Telecommunications, Chongqing, 400065, China

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ABSTRACT

Multi-ary n-cubes are a class of communication patterns that are employed by a number of typical parallel algorithms. This paper addresses the implementation of parallel algorithms with bidirectional and unidirectional ternary n-cube communication patterns on a mesh WDM optical network. For each of these two communication patterns, a routing and wavelength assignment scheme is described, and the number of wavelengths required is shown to attain the minimum, which guarantees the optimality of the proposed scheme.

1. Introduction

Due to extremely high bandwidth, extremely small communication delay and extremely low power consumption, optical networks have been widely regarded as a promising alternative to traditional electrical networks as communication media between processors within parallel computers and, in the past two decades, have received considerable interests from parallel computing community. Wavelength division multiplexing (WDM) technique provides a feasible approach to the realization of optical networks. By dividing the bandwidth of an optical fiber into multiple communication channels represented by their respective wavelengths, a multiplicity of streams of data can be transmitted simultaneously across a same optical fiber. The current WDM technology has already allowed several hundreds of wavelengths per fiber. Moreover, there is anticipation for dramatic progress in future. To eliminate the time cost needed by optoelectrical conversion and processing at intermediate nodes, wavelength converters are not allowed in this paper, so that end-to-end lightpaths are set up between each pair of source–destination nodes. Therefore, a data stream is transmitted in the form of light with given wavelength throughout the transmission process.

To efficiently execute parallel algorithms on WDM networks, it is required to solve the routing and wavelength assignment (RWA) problem, which is intended, for a given WDM network and a parallel algorithm with given communication pattern, to propose a wavelength assignment scheme, i.e., a scheme of assigning a lightpath for each source–destination node pair, for the execution of the parallel algorithm on the WDM network so that the number of wavelengths needed is minimized. In this paper, a lightpath must use a same wavelength on all the links along its path from the source to

* Corresponding author at: College of Computer Science, Chongqing University, Chongqing, 400044, China.
E-mail addresses: cqu2010lab@gmail.com, yucui8712@126.com (X. Yang).

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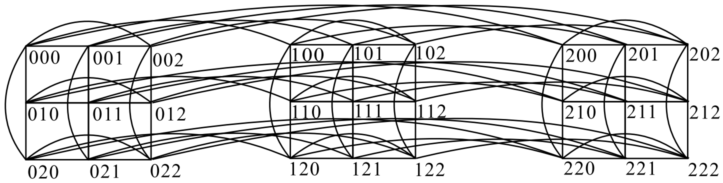

Fig. 1. Ternary 3-cube.

the destination, and all lightpaths sharing a same link (fiber) must be assigned distinct wavelengths. In recent years, the study of the RWA problem for various communication patterns implemented on a variety of WDM networks has received considerable interest [3–5,14–16].

Due to excellent properties such as symmetry, structural recursiveness and high connectivity, on one hand, the k-ary n-cube has been regarded as appealing communication pattern; indeed, quite a number of parallel algorithms have been designed based on k-ary n-cube patterns [6,8,9,12,13]. Mesh topologies, on the other hand, have been considered as promising topologies of WDM networks [4,7,10,11], partly because it is easily implemented. Therefore, the RWA problem for the k-ary n-cube on mesh WDM network is worth study. Very recently, Yu et al. [14] derived an optimal wavelength assignment scheme for the bidirectional ternary n-cube in bidirectional linear array optical networks. To our knowledge, however, the RWA problem for the ternary n-cube on mesh WDM network remains yet to be solved.

This paper addresses the RWA problem for parallel algorithms with ternary n-cube communication pattern on mesh optical network. The main contributions of this paper include: (1) a wavelength assignment scheme for the bidirectional communication pattern is proposed, and its optimality in terms of the number of wavelengths is proved; and (2) a wavelength assignment strategy for the unidirectional communication pattern is designed, and its optimality is shown.

The materials of this paper are organized as follows. Section 2 introduces some preliminaries. In Section 3, a wavelength assignment scheme embedding for bidirectional ternary n-cube is described, and its optimality is shown. In Section 4, an embedding scheme of bidirectional ternary n-cube in mesh optical network is presented, and its optimality is guaranteed. Finally, Section 5 summarizes this work.

2. Preliminary

First, we will introduce the Lee distance between a pair of vectors. For an n-digit radix k vector $x = (x_{n-1}, x_{n-2}, \dots, x_0)$, its Lee weight is defined as

$$W_L(x) := \sum_{i=0}^{n-1} |x_i|, \quad \text{where } |x_i| = \min(x_i, k - x_i).$$

The Lee distance between two vectors, x and y, is denoted as $D_L(x, y) := W_L(x - y)$, which is the Lee weight of their bitwise difference, mod k [9].

Definition 1. The k-ary n-cube, Q_n^k ($k \geqslant 2$ and $n \geqslant 1$), has $N = k^n$ vertices, each of which is labeled with a distinct n-digit radix k vector $x = (x_{n-1}, x_{n-2}, \dots, x_0)$. Two vertices $x = (x_{n-1}, x_{n-2}, \dots, x_0)$ and $y = (y_{n-1}, y_{n-2}, \dots, y_0)$ in Q_n^k are adjacent if and only if $D_L(x, y) = 1$. An edge of Q_n^k with the addresses of the two endpoints differing in the *ith* position is said to be an edge of dimension i.

Fig. 1 depicts a ternary 3-cube (Q_n^3).

The bidirectional ternary n-cube, $Q_n^{3,b}$, is a bidirectional orientation of Q_n^3. Let $DIM_{n,i}^b$ denote the set of all directed edges of dimension i in $Q_n^{3,b}$. Then

$$E\big(Q_n^{3,b}\big) = \bigcup_{i=0}^{n-1} DIM_{n,i}^b$$

$$DIM_{n,i}^b = \big\{\langle j, j+3^i\rangle, \langle j+3^i, j\rangle, \langle j, j+2\times 3^i\rangle, \\ \langle j+2\times 3^i, j\rangle, \langle j+3^i, j+2\times 3^i\rangle, \langle j+2\times 3^i, j+3^i\rangle \,\big|\, j \bmod 3^{i+1} < 3^i\big\}.$$

The unidirectional ternary n-cube, $Q_n^{3,u}$, is a unidirectional orientation of Q_n^3 such that $\langle x, y\rangle$ is a directed edge of dimension i if and only if $y_i = (x_i + 1) \bmod 3$, and $y_j = x_j$ ($j \neq i$ and $0 \leqslant j \leqslant n-1$), where $x = (x_{n-1}, x_{n-2}, \dots, x_1, x_0)$, $y = (y_{n-1}, y_{n-2}, \dots, y_1, y_0)$. Let $DIM_{n,i}^u$ denote the set of all directed edges of dimension i in $Q_n^{3,u}$. Then