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Energy-efficient strategies for building short chains of mobile robots locally ☆

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ABSTRACT

We are given a winding chain of n mobile robots between two stations in the plane, each of them having a limited viewing range. It is only guaranteed that each robot can see its two neighbors in the chain. The goal is to let the robots converge to the line between the stations. The robots are modeled as points in the plane which cannot collide. We use a discrete and synchronous time model, but we restrict the movement of each mobile robot to a distance of δ in each round. This restriction fills the gap between the previously used discrete time model with an unbounded step length and the continuous time model which was introduced in [Bastian Degener, Barbara Kempkes, Peter Kling, Friedhelm Meyer auf der Heide, A continuous, local strategy for constructing a short chain of mobile robots, in: SIROCCO'10: Proceedings of the 17th International Colloquium on Structural Information and Communication Complexity, 2010, pp. 168–182]. We adapt the Go-To-The-Middle strategy by Dynia, Kutylowski, Lorek and Meyer auf der Heide (BICC 2006): In each round, each robot first observes the positions of its neighbors and then moves towards the midpoint between them until it reaches the point or has moved a distance of δ. The main energy consumers in this scenario are the number of observations of positions of neighbors, which equals the number of rounds, and the distance to be traveled by the robots. We analyze the strategy with respect to both quality measures and provide asymptotically tight bounds. We show that the best choice for δ for this strategy is $\delta \in \Theta(\frac{1}{n})$, since this minimizes (up to constant factors) both energy consumers, the number of rounds as well as the maximum traveled distance, at the same time.

1. Introduction

We envision a scenario where two stationary devices (stations) and n mobile robots are placed in the plane. Each mobile robot has two neighbors (mobile robot or station) such that they form a chain between the stations, which can be arbitrarily winding. Assuming that the mobile robots have only a limited viewing range, the goal is to design and analyze strategies for them in order to minimize the length of the chain. Each robot has to base its decision where to move solely on the current position of the neighbors in the chain—no global view, communication or long term memory is provided. The robots are modeled as points in the plane and do not collide.

Our objective is to achieve this goal in an energy-efficient way. The major energy consumers are the energy that is needed to move and the energy that is needed to observe positions of neighboring robots. State of the art work either does not restrict the step length and counts the number of rounds, neglecting the distance that the robots travel, or optimizes the traveled

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Table 1
Results overview.

	1-bounded GTM	δ-bounded GTM	Ccontinuous GTM
Number of time steps	$\Theta(n^2 \log n)$ [2,3]	$\Omega(n^2+\frac{n}{\delta})$ Theorem 2 $\mathcal{O}(n^2 \log n + \frac{n}{\delta})$ Theorem 2	–
Maximum distance	$\Theta(n^2)$ Corollary 2	$\Theta(\delta n^2 + n)$ Theorem 3	$\Theta(n)$ Theorem 5

distance by allowing the robots to observe the positions of neighbors continuously at all times. We want to consider both quality measures at the same time. To the best of our knowledge, we are the first to consider two quality measures at once for a robot formation problem.

As a base we use the Go-To-The-Middle strategy as proposed in [1]. In this intuitive and simple strategy, in each round, each robot computes the current midpoint between its neighbors in the chain and then moves to this point. It is shown that using this strategy, the robots converge to positions on the line between the two stations. It takes $\Theta(n^2 \log n)$ rounds in the worst case until all robots are within distance 1 of the positions they converge to (compare [1] or [2] for the upper bound and [3] for the lower bound). This is also the total number of position measurements per robot. Here, the robots possibly move long distances between two measurements and maybe waste movement energy.

Another approach was taken in [4]. The same problem is investigated, but a continuous time and movement model is used. In this model each robot measures the positions of its neighbors continuously and all the time. It bases its decision in which direction to move on this input. This means that the number of position measurements is infinite. On the other hand, it is shown that using a simple and intuitive strategy called Move-On-Bisector, where each robot always moves in direction of the current bisector of the angle between its neighbors, the robots move at most a distance of $\Theta(n)$ until all robots have reached the line between the stations, which is asymptotically optimal. This model thus makes sense if a robot is able to move and measure positions of neighbors at the same time and if measuring does not take much energy. Since Move-On-Bisector is similar to Go-To-The-Middle, this rises hope that Go-To-The-Middle might also perform well in this model. In this paper, we want to investigate the performance of Go-To-The-Middle in the range between the discrete model with an unlimited step length and the continuous model. We compare the number of position measurements (=number of rounds) and the maximum distance traveled.

In order to trade between the two energy consumers, we introduce a new class of strategies parameterized by a value $\delta \in (0, 1]$, the δ-bounded Go-To-The-Middle strategy (δ-GTM). Here, each robot computes the midpoint between its neighbors and moves towards this point until it is reached or until the robot has moved a distance of δ, whichever occurs first. The Go-To-The-Middle strategy is therefore a special case of δ-GTM with δ set to the viewing range (which we normalize to 1). For δ tending to 0 we get the continuous Go-To-The-Middle strategy (continuous-GTM), which uses the same model as the Move-On-Bisector strategy [4]. In this paper, we analyze δ-GTM thoroughly for any $\delta \in (0, 1]$ with respect to both quality measures and continuous-GTM with respect to the maximum traveled distance (the number of rounds tends to infinity). We provide (almost) tight bounds for their worst case performance. As our main result, we show that for $\delta \in \Theta(\frac{1}{n})$, the combined worst-case energy consumption is asymptotically optimal for δ-GTM: The number of measurements is bounded by $O(n^2 \log n)$ and $\Omega(n^2)$, and the distance traveled is $\Theta(n)$. Thus, no trade-off between the two energy consumers is required. For an overview of the results confer to Table 1.

Related work. Another strategy for the same problem was introduced in [5]. This strategy achieves a linear runtime, but in exchange the robots need to know global coordinates as well as the position of one station. In [6], the more complicated and faster Hopper-strategy is introduced. The idea is to let the robots hop over the midpoint between their two neighbors. This operation combined with the possibility to switch off robots and with Go-To-The-Middle-steps leads to a runtime of $\Theta(n)$ rounds until the sum of the distances between the robots is at most $\sqrt{2}$ times the distance between the stations. The strategy does not guarantee that the robots converge to the line between the stations, but its runtime is asymptotically optimal. For an overview refer to [2].

Similar are robot formation problems: Given n robots in the plane, the goal is to let them build a formation. Considered problems are to let the robots converge to a not-predefined point in the plane (the *Convergence Problem*, [7–9]), to let them gather in such a point (the *Gathering Problem*, [10–15]) or more complex tasks like forming a circle [16,17]. Most of the related work considers very simple robots but with a global view. Exceptions are for example [15,18,19] for the gathering problem and [7,9] for the convergence problem. Moreover, many authors focus on which formations can be achieved in finite time in a given robot and time model, or differently, which robot abilities are crucial to be able to build a formation. Some work also exists with the focus on runtime bounds. [8] gives a runtime bound for the convergence problem of $\mathcal{O}(n^2)$ for halving the length of the convex hull in each dimension. The algorithm uses very simple robots but with a global view. Local algorithms for the gathering problem are analyzed in [18,19].

Organization of the paper. In Section 2 we introduce our model, the strategies which we consider and the two quality measures. Section 3 is devoted to the δ-GTM strategy, with 1-GTM being a special case. In Section 4 we analyze continuous-GTM. Section 3 gives lower and upper bounds for the worst-case number of rounds and both sections for the maximum distance traveled by a robot when using the respective strategy. Finally we give a conclusion and an outlook.