ARTICLE IN PRESS

Computer-Aided Design ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Computer-Aided Design

journal homepage: www.elsevier.com/locate/cad

Hex mesh topological improvement based on frame field and sheet adjustment☆

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ARTICLE INFO

Keywords:
Hex mesh
Mesh optimization
Topological optimization
Frame field
Hex meshing

ABSTRACT

High-quality hex meshes are crucial for finite element analysis. However, geometric smoothing techniques applied for improving the quality of a hex mesh cannot ensure that the requirements of finite element analysis are satisfied. Therefore, to address this challenge, in this paper, we present a novel approach for topological optimization of a hex mesh based on frame field and sheet adjustment. Our approach improves the quality of the worst elements of the hex mesh by optimizing its topological structure. The approach first builds an initial frame field from the input hex mesh and then optimizes it to obtain a resultant high-quality frame field. Next, according to the initial and optimized frame fields, the process determines the most problematic sheet that degrades the quality of the hex mesh. Finally, based on the high-quality frame field and the current topological structure of the hex mesh, our approach uses sheet operations to adjust the structure of this most problematic sheet. Experimental results demonstrate that our topological optimization approach can effectively improve the minimum Scaled Jacobian value of the hex mesh.

1. Introduction

In finite element analysis, the quality of a hex mesh directly determines the precision and efficiency of analysis [1]. A poor-quality element, such as the element whose minimum Scaled Jacobian value is less than 0.2, could make the mesh unusable for the analysis. However, high-quality hex meshing is still a challenging task, and mesh editing can often lead to a degradation in the overall quality of the mesh. Therefore, to adequately perform numerical analysis, hex mesh optimization is an essential step.

Existing hex mesh improvement algorithms can be classified into two categories: geometric smoothing and topological optimization. Geometric smoothing [2], which is the most popular hex mesh improvement method, improves the overall quality of the hex mesh by optimizing the locations of the mesh nodes. However, this technique is unable to modify the topological connections between mesh nodes. Therefore, hex meshes with poor structures cannot be improved through this method. Topological optimization, on the other hand, allows the modification of the connections between the mesh nodes during the optimization process, and is able to provide better improvements to the mesh quality than geometric smoothing for the hex meshes with poor structures. However, due to the global layered structure of hex meshes, topological optimization is a much more complex and difficult process than geometric smoothing, making it challenging to use them in practical scenarios.

Generally, topological modification operations are needed in the process of topological optimization. There are three different types of hex mesh topological modification operations [3]: (a) flipping operations, (b) atomic operations and (c) sheet operations. Similar to the well-known flipping operations in tet meshes, Bern et al. [4] presented flipping operations for hex meshes. The process used patterns for the localization of the hex mesh modifications and translated specific hexahedra into other forms with the same boundary. Tautges et al. [5] presented a set of irreducible atomic operations with the mesh face as the object. Since their works did not consider the global structure of the hex mesh, the topological modification operations proposed by them could not easily improve the quality of a hex mesh. Therefore, in practical scenarios, their operations are rarely used for hex mesh optimization. In sheet operations [6], the layered structure of a hex mesh (called a sheet or Spatial Twist Continuum [7]), is utilized as an operand. Sheet operations change the topology of the hex mesh straightforwardly via the insertion and extraction of sheets, and the adjustment in the direction of the sheet extension.

Based on their objectives, sheet operation based hex mesh topological optimization can be divided into two categories: (a) density optimization [8–10] and (b) valence optimization [11–14]. Density optimization of a hex mesh adds or deletes elements to ensure

☆ This special issue was edited by Scott Canann, Steven J. Owen & Hang Si.

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https://doi.org/10.1016/j.cad.2017.11.007
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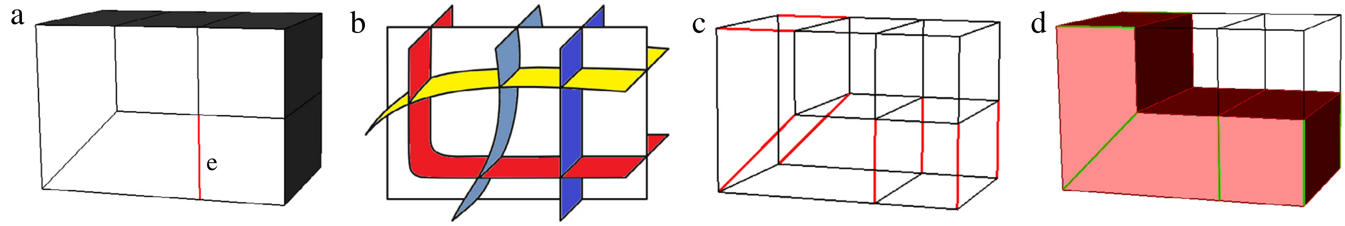

Fig. 1. Illustration of the dual mesh: (a) a hex mesh and an edge *e*; (b) the corresponding dual mesh with five sheets; (c)(d) another two representations of the red sheet in (b). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

that the densities of the elements adhere to the requirements of the density function. In order to coarsen a local region, Woodbury et al. [8] extracted the sheets locally by adding supplementary sheets. Harris et al. [9] presented refinement templates for single and double sheet insertions, to freely control the direction and region of mesh refinement. In order to deal with the problem of uneven density of hex mesh after mesh editing, Zhu et al. [10] proposed sheet operation based coarsening and refinement strategies to optimize the density of the hex mesh. Sheet operations can also be applied to optimize the valences of the mesh edges which can improve the quality of the mesh. Mitchell et al. [11] used the pillowing operation to eliminate the doublet elements of a hex mesh. Shepherd [12] and Qian et al. [13] improved the mesh quality of the boundary elements using the pillowing operation on the entire boundary of a hex mesh. Ledoux et al. [14] used sheet operations to convert a hex mesh into a fundamental mesh. This method improved the quality of the boundary elements along with that of the mesh elements associated with the geometric surfaces or curves. Existing topological optimization algorithms have shown to successfully improve the quality of a hex mesh to different extents. However, there is a need to develop a targeted approach for improving the quality of the worst elements of the hex mesh.

The 3D frame field — 3-tuple vectors defined at every point in 3D space, can offer the ideal geometric orientations on the boundary as well as the inside of the model. Therefore, in recent years, it has been widely used for the generation of high-quality hex meshes [15–18]. Nieser et al. [15] proposed an approach – CubeCover, to generate hex meshes by using the calculated frame field information. Subsequently, several works of research [16–18] were proposed to build an initial frame field and then smooth it to obtain a high-quality frame field, which is crucial for generating high-quality hex meshes. Although there is no theoretical guarantee that the generated frame field will correspond to a topological valid hex mesh, the optimized frame field, to some extent, offers the ideal orientation information of the structure of a hex mesh, even before the mesh generation.

Usually the worst elements in a hex mesh are mainly responsible for the inability of the hex mesh to support successful analysis. Therefore, in order to address this issue, in this paper, we propose an approach for the topological improvement of hex meshes based on frame field and sheet adjustment. Our approach aims to improve the mesh quality of the worst elements in a hex mesh. The main contributions of our proposed approach are listed as follows:

(1) We calculate the structural quality of the relevant sheets of the worst elements to determine the problematic sheet which brings about the worst elements. In this way, we are able to transform the mesh optimization problem into a sheet adjustment problem.
(2) We construct and utilize a high-quality frame field to adjust the structure of the most problematic sheet. This allows us to optimize the topological structure around the worst elements and to improve the mesh quality of these elements.
(3) We present an approach to building a high-quality frame field of the hex mesh, which can effectively guide the topological optimization of the hex mesh.

2. Basic concepts and approach overview

2.1. Basic concepts

Before describing the optimization approach in detail, we introduce some basic concepts regarding the dual structure of the hex mesh and the frame field. For brevity, in this paper, we omit some common concepts about hex meshes, which can be easily found in [19].

2.1.1. Dual mesh

A dual mesh is expressed as the topological structure of a finite element mesh in dual space and consists of a simple arrangement of 2D surfaces. Fig. 1(b) represents the dual mesh of the hex mesh in Fig. 1(a). The elements of a hex mesh have certain relations between them, for example, there exists three groups of topological parallel mesh edges in each element. According to these parallel relationships, all of the edges in a hex mesh can be divided into different groups, with each group corresponding to a sheet in the dual mesh.

Definition 1 (*Sheet*)**.** Given a hex mesh H and a mesh edge e in H, a sheet s can be represented as a set of mesh edges E_s, where E_s is the maximum set of edges that can be recursively found by obtaining the topological parallel edges of e.

Similarly, the set of hexahedra H_s that traverses through E_s and the 2D surface that traverses through the mid points of E_s can also be used to represent the sheet s. The sheet corresponding to the mesh edge e can be represented by the following ways — the red surface in Fig. 1(b); the edge group in red in Fig. 1(c); and the set of hexahedra in Fig. 1(d).

2.1.2. Sheet operations

The basic sheet operations primarily include sheet extraction, sheet inflation and column collapse.

Sheet extraction [3,6] is a dual operation which is used to delete the hexahedra of a sheet. As shown in Fig. 2(b), in the sheet extraction process, the mesh edge of the sheet is degenerated to a point and the hex of the sheet is degenerated to a quad. Using the method in [20], complex sheets including the self-intersecting sheet and self-touching sheet as shown in Fig. 17(e) can also be extracted. The sheet inflation process [3,6] is the inverse operation of sheet extraction. It is a dual operation which inserts a layer of hexahedra to generate a new sheet. In general, sheet inflation takes a quad set as the input and generates the new sheet by inflating every quad in this quad set. As shown in Fig. 2, the sheet in Fig. 2(d) is generated by performing sheet inflation on the quad set in Fig. 2(c). The column collapse operation [3,6] removes the hexahedra in a column to change the intersecting relationship between related