Computer-Aided Design 102 (2018) 182–192

Contents lists available at ScienceDirect

Computer-Aided Design

journal homepage: www.elsevier.com/locate/cad

Discrete multi-material topology optimization under total mass constraint

Xingtong Yang, Ming Li *

State Key Laboratory of CAD&CG, Zhejiang University, Hangzhou, China

ARTICLE INFO

Keywords:
Multi-material
Topology optimization
Discrete solution
Total mass constraint
Theoretical proof

ABSTRACT

A novel approach to computing the discrete solution to the challenging multi-material topology optimization problem under total mass constraint is studied in this paper. The challenge of the problem lies in the incompressibility constraint on the summation of the usage of the total materials, which significantly increases the associated computational difficulty, and is seldom studied before; a few previous studies focus on respective mass constraint on each used material, whose solution lies in a strictly feasible space and is easier to compute. Solution to the optimization problem is derived on a theoretical finding that the iterative density update in a two-material optimization problem is totally determined by the rankings of the elemental compliances, which only involves an FE analysis computation, and can be efficiently achieved. Based on this theoretical insight, a practical *regulated* iterative numerical approach is then devised to find the solution to the multi-material topology optimization problem by solving a series of two-material subproblems. Various 2D and 3D numerical examples demonstrate its capability in providing structure of better compliance as compared with results obtained using latest approach based on density interpolation.

© 2018 Elsevier Ltd. All rights reserved.

1. Introduction

Topology optimization finds the best material distribution within a prescribed design domain, solid or void, in order to produce a structure of optimal performance. Since the seminar work by Bendsoe and Kikuchi [1], topology optimization has undergone a remarkable development over the past decades in both academic research [2–5] and industrial applications [6,7]. Amongst these developments, most of these approaches generally relaxed the problem into a continuous parameter optimization problem taking elemental density (such as SIMP (Solid Isotropic Material with Penalization) [8]) or structural outer shape as design variables (such as level set [9,10]), and then solve it based on the traditional Newton-type (gradient-based) optimization algorithms. Other researches also compute directly discrete solutions to the problem using evolutionary approach (such as BESO (Bi-directional Evolutionary Structural Optimization) [11,12]) or programming techniques [13]. A comprehensive comparison between these approaches are referred to a recent survey [14,3].

In the single-material topology optimization, the material is specified a-priori and the structure is optimized with respect to it. In contrast, the multi-material topology optimization is posed to seek not only the optimal structural form but also simultaneously various material distributions, in order to realize specific design purposes that are otherwise difficult to achieve by single-material structures [15], or to achieve optimal structural performance. As compared with the widely studied former problem, the latter one is much less studied, including approaches based on density (SIMP) [16], phase field [17–19], level set [20], combinatorial optimization [21], or evolutionary approach (BESO) [22]. A recent detailed discussions on the topic are further referred to [19,16].

Challenges. The challenges of multi-material topology optimization are mainly related to its intrinsic mathematical structure of the design space. In the case of single material, the design variable is just the occupation of the single material, whose associated design space is generally sufficiently regular, and can be easily resolved for instance using the optimality criteria [23] or gradient projection [18,19] methods; using discrete variables is only studied by [21]. In contrast, in the case of multi-materials on *total mass* constraint, an additional usage on the summation of the total materials (the incompressibility constraint) is required, which significantly increases the computational costs of the corresponding numerical solution, which is seldom studied before [24–26,21,16].

The different mathematical structures additionally raise challenging issues on an appropriate topology description model, which has to effectively indicate distinct materials inside the domain, fully covering the design domain but not overlapping. Specifically, each elemental domain has a distinct material and they all

* Corresponding author.
E-mail address: liming@cad.zju.edu.cn (M. Li).

https://doi.org/10.1016/j.cad.2018.04.023
0010-4485/© 2018 Elsevier Ltd. All rights reserved.

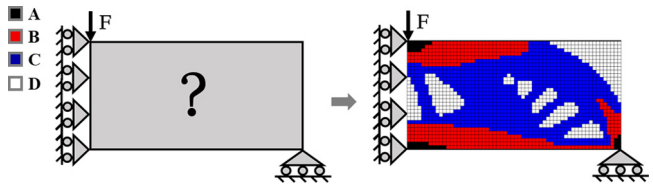

Fig. 1. An example of four-material topology optimization problem. A, B, C and D are four kinds of candidate materials with different Young's modulus and densities.

together, materials and voids, fully cover the design domain. Previous approaches based on density or shape descriptions and interpolation have to carefully devise various strategies to overcome it, and also have a risk of having intermediate design elements with nonphysical materials. The proposed discrete optimization approach naturally avoids these issues.

Approach and contributions. The paper proposes a novel approach to multi-material topology optimization under *total mass* constraint, only studied previously in [21,16] to our best knowledge. The proposed approach is also the first using discrete variables to resolve the multi-material optimization problem under total mass constraint. The discrete variable representation thus avoids the limitations of previous interpolation-based approaches in that they are not physically based or have intermediate design elements. Furthermore, the discrete material expression naturally satisfies the design requirements that all the material densities are separated and fully occupy the design domain.

The overall approach is based on an evolutionary mass reduction strategy, and focuses on the classical compliance minimization problem. The success of the approach is built on a key observation under rigorous proof that the optimal material distribution update in each iteration step only depends on the elemental compliance ranking in case of two materials, and cases of multi-material can be reduced to the two-material problem. The convergence of the overall approach is further improved via introducing a *density regulation* approach that ensures smooth density transition during the optimization process. Performance of the proposed approach is demonstrated through 2D and 3D numerical examples. Its comparisons with results obtained using classical SIMP approach shows its capability in designing structures of better compliance.

The remainder of the paper is arranged as follows. Problem definition and the theoretical basis to resolve it are given in Section 2. Section 3 explains technical details on it numerical implementation. Numerical results on 2D and 3D examples are demonstrated in Section 4, and the paper is concluded in Section 5.

2. Problem and theory

Suppose $\Omega = \{\Omega_e,\ e = 1 \dots N\}$ is a discrete design domain consisting of N disjoint square FE elements $\Omega_e, (E_j, \rho_j),\ j = 1 \dots m$ are the m candidate materials of Young's modulus E_i and density ρ_i satisfying $E_1 \geq \cdots \geq E_m$ and $\rho_1 \geq \cdots \geq \rho_m$; it is assumed that all the base materials have equal Poisson's ratios. Apparently, for candidate materials satisfying $E_i \geq E_j$, $\rho_i \leq \rho_j$, material i is just superior to material j in problem (1), and we can simply choose material i instead of j without further topology optimization.

The multi-material topology optimization problem considers the classical problem of maximizing the stiffness of a structure, or minimizing its compliance under constraint on the usage of total mass, as illustrated in Fig. 1 and stated below. Find the optimal multi-material distribution $\mathbf{x}$ such that

$$\begin{aligned} \min_{\mathbf{x}} \quad & c(\mathbf{x}) = \frac{1}{2}\mathbf{u}^T\mathbf{K}(\mathbf{x})\mathbf{u} \\ s.t. \quad & \mathbf{K}(\mathbf{x})\mathbf{u} = \mathbf{f},\ \mathbf{u} \in \mathcal{U} \\ & M(\mathbf{x}) \leq M^*. \end{aligned} \tag{1}$$

Here $\mathbf{x}$ gives the specific material of every finite element as

$$\mathbf{x} = \{\mathbf{x}_e\},\ \mathbf{x}_e = \{x_{ej}\},\ e = 1, \dots, N,\ j = 1, \dots, m, \tag{2}$$

where $x_{ej} = 1$ or 0 determines whether element e is filled with the jth material at an addition requirement that each element e is filled by one and only one kind of material, that is,

$$\sum_{j=1}^{m} x_{ej} = 1. \tag{3}$$

The total mass is defined as follows

$$M(\mathbf{x}) = \sum_{e=1}^{N} \rho(\mathbf{x}_e),\ \text{for } \rho(\mathbf{x}_e) = \sum_{j=1}^{m} x_{ej}\rho_j. \tag{4}$$

In addition, $\mathbf{u}$ and $\mathcal{U} \subset \mathbb{R}^N$ are the nodal displacement vector and its admissible space, where certain Dirichlet boundary conditions are prescribed. $\mathbf{K}(\mathbf{x})$ is the global stiffness matrix, decided by material distribution $\mathbf{x}$, which is calculated by

$$\mathbf{K}(\mathbf{x}) = \{E(\mathbf{x}_e)\mathbf{k}_0\},\ E(\mathbf{x}_e) = \sum_{j=1}^{m} x_{ej}E_j \tag{5}$$

for a unit stiffness matrix $\mathbf{k}_0$. $\mathbf{f}$ is the external force vector. The structure compliance $c(\mathbf{x})$ is calculated by

$$c(\mathbf{x}) = \frac{1}{2}\mathbf{u}^T\mathbf{K}(\mathbf{x})\mathbf{u} = \sum_{e=1}^{N}\sum_{j=1}^{m} \frac{1}{2}x_{ej}E_j\mathbf{u}_e^T(\mathbf{x})\mathbf{k}_0\mathbf{u}_e(\mathbf{x}), \tag{6}$$

where $\mathbf{u}_e$ is the displacement vector of element e.

The original problem (1) is equivalently written

$$\min_{\mathbf{x}\in\mathcal{A}}\{c(\mathbf{x}) \mid \mathbf{K}(\mathbf{x})\mathbf{u} = \mathbf{f}\}, \tag{7}$$

where the design domain $\mathcal{A} = \{\mathbf{x}|M(\mathbf{x}) \leq M^*\}$.

The proposed approach to resolving problem (1) is achieved via gradually reducing the mass from an initial distribution till the target one, as also used in the well-studied evolutionary approach BESO [27]. The initial value of $\mathbf{x}$ is set as density fully filled by the material M^1 of the largest Young's modulus among all the candidate materials, which is an obvious global optimal solution under the associated mass constraint. Before further explanation on the overall approach, we first explain below the density update strategy in each of the optimization step as the base of the proposed approach.

Noticing that the stiffness matrix $\mathbf{K}$ and displacement $\mathbf{u}$ are both dependent on the density $\mathbf{x}$, taking derivatives on both sides of the equilibrium equation

$$\mathbf{K}(\mathbf{x})\mathbf{u}(\mathbf{x}) = \mathbf{f} \tag{8}$$

with respect to the design variable x_{ej} gives

$$\frac{\partial\mathbf{K}(\mathbf{x})}{\partial x_{ej}}\mathbf{u}(\mathbf{x}) + \mathbf{K}(\mathbf{x})\frac{\partial\mathbf{u}(\mathbf{x})}{\partial x_{ej}} = 0, \tag{9}$$

and by basic transformation, there is

$$\frac{\partial\mathbf{u}(\mathbf{x})}{\partial x_{ej}} = -\mathbf{K}^{-1}(\mathbf{x})\frac{\partial\mathbf{K}(\mathbf{x})}{\partial x_{ej}}\mathbf{u}(\mathbf{x}). \tag{10}$$

On the other hand, according to the compliance definition and equilibrium equation (8)

$$c(\mathbf{x}) = \frac{1}{2}\mathbf{u}^T\mathbf{K}(\mathbf{x})\mathbf{u}(\mathbf{x}) = \frac{1}{2}\mathbf{u}^T(\mathbf{x})\mathbf{f}, \tag{11}$$

we have its partial derivative with respect to design variable x_{ej}

$$\frac{\partial c(\mathbf{x})}{\partial x_{ej}} = \frac{1}{2}\mathbf{f}^T\frac{\partial\mathbf{u}}{\partial x_{ej}}, \tag{12}$$

Download English Version:

https://daneshyari.com/en/article/6876404

Download Persian Version:

https://daneshyari.com/article/6876404

Daneshyari.com