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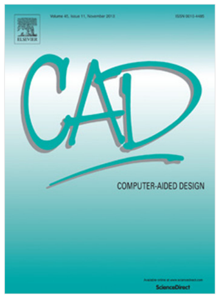

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Model reduction in geometric tolerancing by polytopes

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Abstract

There are several models used in mechanical design to study the behavior of mechanical systems involving geometric variations. By simulating defects with sets of constraints it is possible to study simultaneously all the configurations of mechanisms, whether over-constrained or not. Using this method, the accumulation of defects is calculated by summing sets of constraints derived from features (toleranced surfaces and joints) in the tolerance chain. These sets are usually unbounded objects ($\mathbb{R}^6$-polyhedra, 3 parameters for the small rotation, 3 for the small translation), due to the unbounded displacements associated with the degrees of freedom of features. For computational and algorithmic reasons, cap facets are introduced into the operand polyhedra to obtain bounded objects ($\mathbb{R}^6$-polytopes) and facilitate computations. However, the consequence is an increase in the complexity of the models due to the multiplication of caps during the computations. In response to this situation, we formalized and tested a method for controlling the effects of cap facets. Based on the combinatorial properties of polytopes, we propose to trace the operand faces during the different operations. An industrial case is solved and discussed in order to show the significant gain in computational time when applying the new method. This example has been chosen to be as general as possible to illustrate the genericity of the method.

Keywords: Tolerance analysis, Degrees of freedom, Polytopes, Traceability, Cap half-space, Minkowski sum

1. Introduction

In mechanical design, a tolerance zone represents the limits of manufacturing defects for a given surface. When the surface is considered as a discrete set of points, this restriction is transferred from each point to a given 3D point M. This point is assumed rigidly linked to the toleranced surface. Considering manufacturing defects as small displacements [1], each constraint can be modelled as a half-space in the 6-dimensional space of deviations [2]:

$$\bar{H}_k^+ = \left\{ \boldsymbol{x} \in \mathbb{R}^6 : b_k + a_{k_1} x_1 + ... + a_{k_6} x_6 \geq 0 \right\} \quad (1)$$

where x_1, x_2, x_3 are the rotation variables, x_4, x_5, x_6 are the translation variables, the second member b_k is related to the size of tolerance zone and a_{k_j} $(1 \leq j \leq 6)$ are scalar parameters dependent on the geometry of the toleranced feature and the location of the point M where the constraints are defined.

When a set of m points is considered, a set of $k_{max} = 2m$ half-spaces is obtained. They, in turn, define a convex $\mathcal{H}$-polyhedron (where $\mathcal{H}$ stands for half-space) in $\mathbb{R}^6$ [2]:

$$\Gamma = \bigcap_{k=1}^{k_{max}} \bar{H}_k^+ \quad (2)$$

Let us take, for example, the toleranced surface depicted at the left-hand side in Figure 1. For illustrative purposes, let us consider it as a 2D model: only the rotation along the z-axis and the translations along the x-axis and the y-axis (hereafter r_z, t_x and t_y respectively) are taken into account. The restriction on position imposed by the tolerance

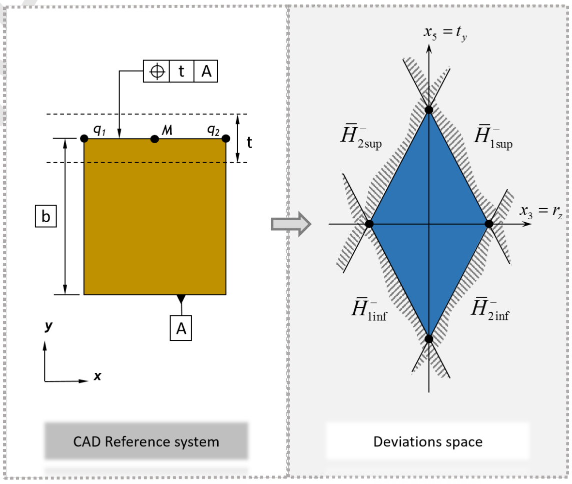

Figure 1: From a toleranced feature in the CAD reference system (left) to its respective polytope in the deviations space (right) considering the constraints expressed at point M.

zone on the toleranced feature can be modelled with four half-spaces. For point q_1, the half-spaces $\{\bar{H}_{1sup}^+,\ \bar{H}_{1inf}^+\}$ are obtained. Similarly, $\{\bar{H}_{2sup}^+,\ \bar{H}_{2inf}^+\}$ are derived from the restriction of q_2. These constraints are expressed at point M (see Figure 1). In the space spanned by $[r_z, t_y]$ the intersection of these half-spaces defines a bounded polyhedron, i.e. a polytope (see right-hand side of Figure 1). As the tolerance zone does not impose limits on t_x, the intersection of the half-spaces generates an unbounded object in the 3D space defined by $[r_z, t_x, t_y]$.

Similarly, the displacements of a couple of surfaces potentially in contact can be modelled with a polyhedron.