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Hexahedral mesh smoothing via local element regularization and global mesh optimization[☆]

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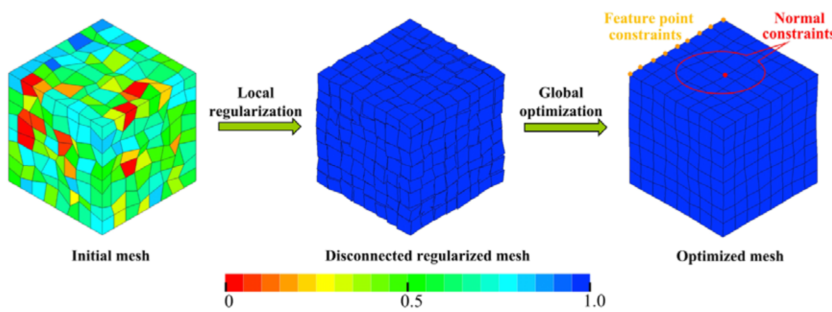
HIGHLIGHTS

- A novel local to global hexahedral mesh smoothing algorithm is proposed.
- An element size adjustment method is proposed to scale transformed ideal elements.
- The volumetric Laplacian operator is used to stitch the regularized elements.
- Geometric constraints of surface meshes are introduced to global optimization.

GRAPHICAL ABSTRACT



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ABSTRACT

The quality of finite element meshes is one of the key factors that affect the accuracy and reliability of finite element analysis results. In order to improve the quality of hexahedral meshes, we present a novel hexahedral mesh smoothing algorithm which combines a local regularization for each hexahedral mesh, using dual element based geometric transformation, with a global optimization operator for all hexahedral meshes. The global optimization operator is composed of three main terms, including the volumetric Laplacian operator of hexahedral meshes and the geometric constraints of surface meshes which keep the volumetric details and the surface details, and another is the transformed node displacements condition which maintains the regularity of all elements. The global optimization operator is formulated as a quadratic optimization problem, which is easily solved by solving a sparse linear system. Several experimental results are presented to demonstrate that our method obtains higher quality results than other state-of-the-art approaches.

1. Introduction

Hexahedral meshes are preferred to tetrahedral meshes in many finite element analysis applications, since hexahedral meshes offer several numerical advantages over tetrahedral meshes due to their tensor product nature [1]. However, the automatic hexahedral mesh generation for a complicated three-dimensional solid remains far more challenging than the generation of tetrahedral meshes. Many hexahedral meshes are generated by the divide and conquer approach that decomposes the entire geometry into several simpler volumes [2], which can be subsequently meshed by using mapping/sub-mapping [3,4], sweeping [5,6], and advancing front method [7,8]. However, for solid models with complex geometries, hexahedral meshing frequently produces meshes of unsatisfactory quality. As accuracy and reliability of finite element analysis results depend on mesh

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quality, there is frequently a need to improve the quality of hexahedral meshes.

Hexahedral mesh quality improvement involves two main approaches: node-smoothing method and topological modification method [9,10]. The topological modification method is used for hexahedral meshes which have poor topological connections with neighboring meshes. In most cases, topological modifications of hexahedral meshes are far more complex than other types of meshes due to their specific adjacency structure. Therefore, we focus on node-smoothing method, which can improve mesh quality only based on node movements without altering mesh topology.

1.1. Previous work

Much work has been done on hexahedral mesh smoothing, and the related techniques can be classified by the general strategies they employ, including local smoothing and global smoothing. While local smoothing methods operate one vertex at a time to improve mesh quality in a neighborhood of that vertex, global smoothing methods aim to improve the overall mesh quality by solving a linear or non-linear system. Laplacian smoothing method is considered as the most well-known local smoothing method for mesh smoothing [11]. In Laplacian smoothing, nodes are iteratively moved to the geometric center by weighting the contribution of each neighboring node in the averaging function. While Laplacian method is simple and easy to use, it may produce invalid elements and is not effective in models which have non-convex regions. In order to ensure the mesh quality improvement and to prevent the generation of invalid elements, a smart variant of Laplacian smoothing is proposed, where node moves only if the quality of the local mesh is improved [12]. Freitag and Plassmann [13] presented a local optimization method for mesh quality improvement which aims to maximize the sum of minimal angle of local influence mesh, and new node positions can be obtained by solving a non-linear system using an analogue of the steepest descent method. Instead of mesh quality, a node quality metric is chosen in [14] and a combined gradient driven and simulated annealing technique is used to force each node to a better position. Zhang et al. [15] proposed a feature preserving mesh quality improvement method, which moves each surface node in their tangent plane and relocates every inner node to the mass center of all its surrounding hexahedra to improve the aspect ratio. Vartziotis and Wipper [16,17] proposed a geometric element transformation method (GETMe) for hexahedral mesh smoothing based on a dual element based regularizing transformation, in which low quality elements are sequentially transformed and new inner node positions are updated by averaging the transformed element nodes only if quality is improved. While local smoothing is simple and computationally efficient, it cannot guarantee the quality improvement of overall mesh.

The most widely used global smoothing is global optimization-based method. In global optimization-based method, mesh quality improvement is performed by the minimization of a smoothed objective function, which is established based on different mesh quality metrics. The existing methods differ mainly in the quality metrics and minimization techniques used. In [18], the condition number of Jacobian matrix is chosen to create global objective functions, and smoothing results can be obtained by conjugate gradient and line-search method. With the same objective function, Yilmaz and Kuzuoglu [19] proposed a particle swarm approach based smoothing method to improve the overall mesh quality. Guerra [20] presented a modified objective function of hexahedral meshes that depends on the shape matrix and a function used in tetrahedral mesh smoothing, and the global minimum with this modified objective function can be reached by Newton–Raphson method. While global optimization-based methods yield better results, they are computationally expensive and non-convergence occurs in some cases. In [21], a physics-based smoothing method is used to improve the overall mesh quality, in which hexahedral meshes are deformed by resolving a linear system composed of global stiffness matrix. Physics-based smoothing method provides good results while it is only applicable to hexahedral meshes without poor quality. In addition, the space mapping technique is introduced to the improvement of the quality of tetrahedral and hexahedral meshes [22].

1.2. Contributions

While hexahedral mesh smoothing methods considered only local smoothing or global smoothing, the smoothing operations often led to invalid quality improvements and non-convergence results. Additionally, most of hexahedral mesh smoothing methods did not take into account the effect of surface mesh quality on smoothing results, since the quality of surface meshes is one of the key factors that affect the improvement of the minimal element quality. In this paper, we proposed a local to global hexahedral mesh smoothing algorithm using the volumetric Laplacian. Instead of a local or global smoothing technique, our smoothing algorithm consists of geometric transformation based local regularization and global optimization using the volumetric Laplacian operator. The global optimization procedure is performed by the minimization of a quadric energy function, which is composed of volumetric Laplacian operator, transformed node displacements and geometric constraints of surface meshes.

Contributions of this paper are summarized as follows:

- An element size adjustment algorithm based on edge direction constraints and node position constraints is proposed to scale ideal elements produced by geometric transformation.
- The global volumetric Laplacian operator is used to stitch the regularized elements that can ensure the validity of smoothed hexahedral meshes.
- Geometric constraints of surface meshes are introduced to global optimization of hexahedral meshes that can highly improve the minimal quality of hexahedral meshes.

2. Hexahedral mesh smoothing algorithm

Our hexahedral mesh smoothing algorithm is based on a local to global smoothing strategy, which consists of local hexahedron regularization and global optimization. Each individual hexahedral element is locally regularized by dual element based geometric transformation method. The geometric transformation has a strong regularizing effect, and meanwhile the size of the hexahedral element might also changes significantly. The size of the ideal element obtained by geometric transformation must be adjusted so that the regularized element keeps the size close to the initial one. In order to stitch the regularized elements together, we adopt the volumetric Laplacian operator in a global way and surface geometric constraints are also introduced to preserve the features of the original hexahedral meshes. The steps of our algorithm are described as follows:

Step 1: Local regularization of hexahedral meshes

For each element of the input hexahedral meshes, performing the following steps:

Step 1.1 Transform the element to its corresponding ideal element by using dual element based geometric transformation method.