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Brain morphometry on congenital hand deformities based on Teichmüller space theory☆

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HIGHLIGHTS

- We propose a novel Teichmüller space theory approach to study brain morphometry.
- Conformal welding signature reflects the geometric relations of different regions.
- The invertible method encodes complete information of the functional area boundaries.
- We evaluate signatures of pre-central and post-central gyrus on subjects and control.
- Congenital Hand Deformities may make a greater impact on post-central gyrus.

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ABSTRACT

Congenital Hand Deformities (CHD) usually occurred between the fourth and the eighth week after the embryo is formed. Failure of the transformation from arm bud cells to upper limb can lead to an abnormal appearing/functioning upper extremity which is presented at birth. Some causes are linked to genetics while others are affected by the environment, and the rest have remained unknown. CHD patients develop prehension through the use of their hands, which affects the brain as time passes. In recent years, CHD have gained increasing attention and researches have been conducted on CHD, both surgically and psychologically. However, the impacts of CHD on the brain structure are not well-understood so far. Here, we propose a novel approach to apply Teichmüller space theory and conformal welding method to study brain morphometry in CHD patients. Conformal welding signature reflects the geometric relations among different functional areas on the cortex surface, which is intrinsic to the Riemannian metric, invariant under conformal deformation, and encodes complete information of the functional area boundaries. The computational algorithm is based on discrete surface Ricci flow, which has theoretic guarantees for the existence and uniqueness of the solutions. In practice, discrete Ricci flow is equivalent to a convex optimization problem, therefore has high numerically stability. In this paper, we compute the signatures of contours on general 3D surfaces with the surface Ricci flow method, which encodes both global and local surface contour information. Then we evaluated the signatures of pre-central and post-central gyrus on healthy control and CHD subjects for analyzing brain cortical morphometry. Preliminary experimental results from 3D MRI data of CHD/control data demonstrate the effectiveness of our method. The statistical comparison between left and right brain gives us a better understanding on brain morphometry of subjects with Congenital Hand Deformities, in particular, missing the distal part of the upper limb.

1. Introduction

Congenital anomalies affect 1%–2% of newborns, and approximately 10% of those children have hand abnormalities [1,2]. Congenital hand deformities (CHD) usually occurred between the fourth and the eighth week after the embryo is formed [3]. Failure of the transformation from arm bud cells to upper limb can

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result in an abnormal appearing/functioning upper extremity which is presented at birth. Some causes are linked to genetics while others are affected by the environment—either outside or inside the uterus, and the rest has remained unknown.

CHD are present at birth and may become a challenge for children as they continue to grow and learn to interact with their environment through the use of their hands. The degree of deformity can vary from something minor, such as a digital disproportion, to a more severe deformity, such as total absence of a bone or part of the hand. Children develop prehension with hands as they are, and they usually start being self-conscious of the difference when they socialize in school, as a result, they may intend not to show or use it. The less the affected arm and hand are used, the less functional they will be, and the more likely it will affect functionalities of brain. In recent years, a number of researches have been conducted on mechanism, behaviors and psychology for CHD [4–7]. However, the impacts of CHD on brain structure are not well-understood so far.

Most brain MRI scanning protocols acquire volumetric data about the anatomy of the subject. Early researches [8,9] have demonstrated that surface-based brain mapping may offer advantages over volume-based brain mapping work [10] to study structural features of the brain, such as cortical gray matter thickness, complexity, and patterns of brain change over time due to disease or developmental processes. In research studies that analyze brain morphology, many surface-based shape analysis methods have been proposed, such as spherical harmonic analysis (SPHARM) [11,12], minimum description length approaches [13], medial representations (M-reps) [14], cortical gyrification index [15], shape space [16], metamorphosis [17], momentum maps [18] and conformal invariants [19]; these methods may be applied to analyze shape changes or abnormalities in cortical and subcortical brain structures. Even so, a stable method to compute a global intrinsic transformation-invariant shape descriptors would be highly advantageous in this research field.

Here, we propose a novel and intrinsic method to compute the global correlations between various surface region contours in Teichmüller space and apply it to study brain morphology on CHD. The proposed shape signature demonstrates the global geometric features encoded in the interested regions, as a biomarker for measurements of CHD progression and pathology. It is based on the brain surface conformal structure [20–23] and can be accurately computed using the surface Ricci flow method [24,25].

1.1. Our approach

For a 3D surface, all the contours represent the 'shape' of the surface. Inspired by the research work of Sharon and Mumford [26] on 2D shape analysis (recently it has been generalized to model multiple 2D contours [27]), we build a Teichmüller space for 3D shapes by using conformal mappings. In this Teichmüller space, every 3D contour (a simple closed curve) is represented by a point in the space; each point denotes a unique equivalence class of diffeomorphisms up to a Möbius transformation. For a 3D surface, the diffeomorphisms of all the contours form a global shape representation of the surface. By using this signature, the similarities of 3D shapes can be quantitatively analyzed, therefore, the classification and recognition of 3D objects can be performed from their observed contours.

We tested our algorithm in segmented regions on a set of brain left cortical surfaces extracted from 3D anatomical brain MRI scans. The proposed method can reliably compute signatures on two cortical functional areas [28] by computing the diffeomorphisms of each observed contour. Using the signature as the statistics, our method achieves about 91% accuracy rate to discriminate a set of CHD subjects from healthy control subjects.

To the best of our knowledge, it is the first work to apply contour diffeomorphism to brain morphometry research. Our experimental results demonstrated that this novel and simple method may be useful to analyze certain functional areas, and it may shed some light on understanding and detecting abnormality regions in brain surface morphometry. Our major *contributions* in this work include:

(i) A new method to compute Teichmüller shape descriptor, in a way that generalized a prior 2D domain conformal mapping work [26].
(ii) The method is theoretically rigorous and general. It presents a stable way to calculate the diffeomorphisms of contours in general 3D surfaces based on Ricci flow.
(iii) It involves solving elliptic partial differential equations (PDEs), so it is numerically efficient and computationally stable.
(iv) The shape descriptors are global and invariant to rigid motion and conformal deformations.

Pipeline. Fig. 1 shows the pipeline for computing the diffeomorphism signature for a surface with 4 closed contours. Here, we use a human brain hemisphere surface whose functional areas are divided and labeled in different colors. The contours (simple closed curves) of functional areas can be used to slice the surface open to connected patches. As shown in frames (a, b, g), four contours $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ are used to divide the whole brain (a genus zero surface S) to 5 patches S_0, S_1, S_2, S_3, S_4; S_0 is a genus zero surface with four boundaries and thus could be conformally mapped to a disk with one exterior circle and three interior circles, as shown in frame (c). In order to study the correlation of left and right functional areas with respect to the corresponding half brain, we have cut the whole brain surface into two halves, S_{left} and S_{right} respectively. Each of S_1, S_2, S_3, S_4 is conformally mapped to a circle domain (e.g., disk or annuli), D_1, D_2, D_3, D_4, in (d). Each of C_1, C_2, C_3, C_4 in frame (e) is mapped from $(S_{left} - S_1)$, $(S_{left} - S_2)$, $(S_{right} - S_3)$, $(S_{right} - S_4)$, the complement of S_1, S_2, S_3, S_4 with respect to the corresponding half brain. One contour γ_i, $(i = 1, 2, 3, 4)$ is mapped to two unit circles in two mappings, which are boundaries of two topological disks, D_i^{in} and D_i^{out}. Technically, outer topological disks D_i^{out} are mapped from a topological annulus C_i, frame (e). The inner boundary of the annulus forms the circle, while the outer boundary represents the connection between the left and the right half of the brain. The representation of the shape according to each contour is a diffeomorphism of the unit circle to itself, defined as the mapping between periodic polar angles $(Angle_{in}, Angle_{out})$, $Angle_{in}, Angle_{out} \in [0, 2\pi]$, which is determined only by the target functional area and the corresponding half brain surface. The proper normalization is employed to remove Möbius ambiguity. The diffeomorphisms induced by the conformal maps of each curve form a diffeomorphism signature, which is the Teichmüller coordinates in Teichmüller space. As shown in (f, h), the curves demonstrate the diffeomorphisms for two contours; the l_2 norm of the area distance is defined as the metric for shape comparison and classification.

2. Theoretic background

In this section, we briefly introduce the theoretical foundations necessary for the current work. For more details, we refer readers to the classical books [29,30].

2.1. Surface uniformization mapping

Conformal mapping between two surfaces preserves angles. Suppose $(S_1, \mathbf{g}_1)$ and $(S_2, \mathbf{g}_2)$ are two surfaces embedded in $\mathbb{R}^3$, $\mathbf{g}_1$ and $\mathbf{g}_2$ are the induced Euclidean metrics.