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On modeling with rational ringed surfaces☆

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HIGHLIGHTS

- The paper focuses on modeling with rational ringed surfaces, mainly for blending purposes.
- We answer the question of their rationality and use P-curves for constructing rational ringed surfaces.
- The method for constructing blends that satisfy certain prescribed constraints is presented.
- The designed approach can be easily modified also for computing n-way blends.
- The contour curves are used for computing approximate parameterizations of implicitly given blends by ringed surfaces.

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ABSTRACT

A surface in Euclidean space is called ringed (or cyclic) if there exists a one-parameter family of planes that intersects this surface in circles. Well-known examples of ringed surfaces are the surfaces of revolution, (not only rotational) quadrics, canal surfaces, or Darboux cyclides. This paper focuses on modeling with rational ringed surfaces, mainly for blending purposes. We will deal with the question of rationality of ringed surfaces and discuss the usefulness of the so called P-curves for constructing rational ringed-surface-blends. The method of constructing blending surfaces that satisfy certain prescribed constraints, e.g. a necessity to avoid some obstacles, will be presented. The designed approach can be easily modified also for computing n-way blends. In addition, we will study the contour curves on ringed surfaces and use them for computing approximate parameterizations of implicitly given blends by ringed surfaces. The designed techniques and their implementations are verified on several examples.

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1. Introduction

In this paper, we will focus on modeling with the *ringed* (or *cyclic*) *surfaces*. They are generated by sweeping a circle with variable radius – which is contained in a plane with a possibly non-constant normal vector – along a directrix curve, see [1]. Special cases include *Darboux cyclides* (see [2]) which can carry up to six families of real circles, see [3–6]. Ringed surfaces are well suited e.g. for designing pipe structures in plant modeling, [7,8]. A special method for computing the intersection of two ringed surfaces is described in [9].

The class of ringed surfaces contains a special subclass of *canal surfaces* that are obtained as the envelopes of a one-parameter family of spheres, see e.g. [10,11]. Special cases include *pipe surfaces* (obtained for spheres of constant radius) and *surfaces of revolution* (generated by spheres whose centers are located on a given line). *Dupin cyclides* (defined as the envelopes of all spheres touching three given spheres) form probably the most studied family of canal surfaces [12–14].

It was proved in [15,16] that any canal surface with a rational spine curve (which is the curve formed by the centers of the moving spheres) and a rational radius function admits a rational parameterization [15,16]. An algorithm for generating rational parameterizations of canal surfaces was presented in [17]. An analogous result was proved also for normal ringed surfaces—in particular a normal ringed surface with rational directrix and a rational radius function is rational, see [15]. We will extend this result for an arbitrary ringed surface with rational directrix, rational normal vector field and radius function equal to the square root of some non-negative rational function.

The canal surfaces have been studied more thoroughly than the ringed surfaces and one can find many applications in CAD based on canal surfaces. Let us recall, for instance, the operation of blending which is one of the most important operations in

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Computer-Aided (Geometric) Design. The main purpose of this operation is to generate one or more surfaces that create a smooth joint between the given shapes. Blending surfaces are necessary for rounding edges and corners of mechanical parts, or for smooth connection of separated objects, see e.g. [18,19] and references therein. Canal surfaces are often used as blending surfaces between two given surfaces [20–22].

However not for all input shapes (ending with circular profiles), the canal surfaces are suitable as blending primitives. We recall e.g. the case when oblique cones shall be blended—then the ringed surfaces find their straightforward application. Our goal is to investigate the possibility to construct a smooth joint between given shapes by parametric ringed surfaces. In addition, the designed technique will enable us to construct also general n-way blends between n ringed surfaces. As the method produces watertight blends the results can be interesting also for Isogeometric Analysis, [23].

Methods for connecting ringed surfaces by implicit blends were presented in [24,25]. As known, an exact rational parameterization does not exist for an arbitrary implicitly given algebraic surface. Then, suitable techniques producing approximate parameterizations must be used. Methods for the computation of an approximate parameterization of a implicitly given canal surface obtained as a result of some blending technique were thoroughly studied in [26,27]. In this paper, we will continue with this approach and present a method for an approximate parameterization of implicitly given ringed-surface-blends.

The remainder of this paper is organized as follows. The next section summarizes some fundamental facts about ringed surfaces. In Section 3 we deal with the question of rationality of ringed surfaces. The method of constructing blending surfaces by rational ringed surfaces is described in Section 4. In Section 5 we show how such a blending surface can be adjusted to satisfy certain constraints, e.g. when avoiding obstacles is required. In Section 6 we present a short sketch of the construction of n-way blends and Section 6 is devoted to contour curves on ringed surfaces which are used for computing approximate parameterizations of implicitly given blends by ringed surfaces. Then we conclude the paper.

2. Preliminaries

A *ringed surface* $\mathcal{R}$, see Fig. 1, is a surface generated by sweeping a circle centered at a curve $\mathbf{d}(t)$, called the *directrix*, lying in a plane with the prescribed normal vector $\mathbf{n}(t)$, called the *orientation function*, and possessing a radius described by the *radius function* $\varrho(t)$. We shortly write $\mathcal{R} : (\mathbf{d}, \mathbf{n}, \varrho)(t)$. We will omit the dependence on parameter t whenever no confusion is likely to arise. When $\mathbf{n} \times \mathbf{d}' = \mathbf{o}$ (i.e., the orientation function describes the field of the directrix tangents' directions) we speak about the *normal ringed surfaces*, see Fig. 2.

A special subclass of the ringed surfaces is constituted by the so called *canal surfaces*. A canal surface is defined as the envelope of a one-parameter family of spheres whose centers trace the *spine curve* $\mathbf{m}$ and possess the radii described by the *radius function* r. The canal surfaces with constant radius function are called *pipe surfaces*.

Every canal surface is a ringed surfaces with

$$\mathbf{d} = \mathbf{m} - \frac{rr'}{\|\mathbf{m}'\|^2}\mathbf{m}', \qquad \mathbf{n} = \mathbf{m}', \qquad \varrho = \frac{r\sqrt{\|\mathbf{m}'\|^2 - r'^2}}{\|\mathbf{m}'\|}. \tag{1}$$

On the other hand a ringed surface is a canal surface if and only if

$$\left(\left(\mathbf{n} \cdot \mathbf{d}'\right)\mathbf{d}' + \left(\varrho\varrho'\right)\mathbf{n}'\right) \times \mathbf{n} \equiv \mathbf{o}, \tag{2}$$

see [6], where a close relation of ringed/canal surfaces to Darboux cyclides is investigated.

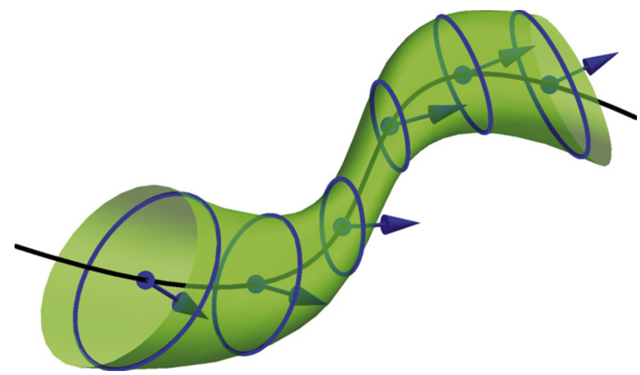

Fig. 1. A ringed surface (green) with a directrix (black), vectors corresponding to the orientation function (blue) and sweeping circles (blue). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

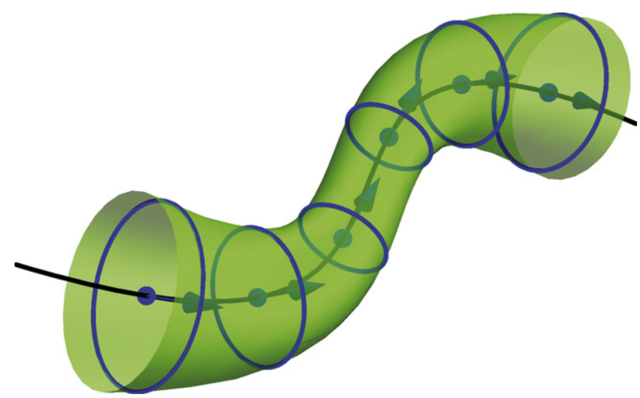

Fig. 2. A normal ringed surface (green) with a directrix (black), vectors corresponding to the orientation function (blue) and sweeping circles (blue). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

A parameterization $\mathbf{s}(t, u)$ of the ringed surface $\mathcal{R}$ can be obtained by rotating the points of a (suitably chosen) curve $\mathbf{c}(t)$ on $\mathcal{R}$ (different from any of the sweeping circles) around the corresponding line element $\big(\mathbf{d}(t), \mathbf{n}(t)\big)$, i.e., we arrive at

$$\mathbf{s}(t, u) = \mathbf{d} + \frac{(\varphi + \mathbf{n}) \star (\mathbf{c} - \mathbf{d}) \star (\varphi - \mathbf{n})}{(\varphi + \mathbf{n}) \star (\varphi - \mathbf{n})}, \tag{3}$$

where $\varphi(u)$ is a rational function (for the sake of simplicity we choose $\varphi(u) = u$ for the low rational degree of $\mathbf{s}(t, u)$ in u), the sums $\varphi(u) \pm \mathbf{n}(t)$ of scalars and vectors are considered as quaternions, and $\star$ is the operation of quaternion multiplication, see e.g. [28] for more details about quaternions.

Hence the problem of finding a rational parameterization of $\mathcal{R}$ is reduced to the problem of finding a rational curve $\mathbf{c} \subset \mathcal{R}$. All such curves (except from the sweeping circles) can be written in the form

$$\mathbf{c} = \mathbf{d} + \varrho \frac{\mathbf{n}^{\perp}}{\|\mathbf{n}^{\perp}\|}, \tag{4}$$

where $\mathbf{n}^{\perp}$ is a vector field perpendicular to the vector field $\mathbf{n}$.

3. Rationality of ringed surfaces

Here, we will consider the ringed surface $\mathcal{R} : (\mathbf{d}, \mathbf{n}, \varrho)$ given by a rational directrix $\mathbf{d}$, rational radius function ϱ and rational orientation function $\mathbf{n}$. As discussed above, to compute a rational

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