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Isogeometric segmentation: The case of contractible solids without non-convex edges

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HIGHLIGHTS

- Segmentation of a 3D solid without non-convex edges into topological hexahedra.
- Method is based on the edge graph of the solid.
- Decomposition is done by means of simple combinatorial and geometric criteria.
- Number of resulting topological hexahedra is small.

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ABSTRACT

We present a novel technique for segmenting a three-dimensional solid with a 3-vertex-connected edge graph consisting of only convex edges into a collection of topological hexahedra. Our method is based on the edge graph, which is defined by the sharp edges between the boundary surfaces of the solid. We repeatedly decompose the solid into smaller solids until all of them belong to a certain class of predefined base solids. The splitting step of the algorithm is based on simple combinatorial and geometric criteria. The segmentation technique described in the paper is part of a process pipeline for solving the isogeometric segmentation problem that we outline in the paper.

1. Introduction

Isogeometric analysis (IGA) is a novel framework for numerical simulation that often relies on a NURBS volume representation of the computational domain. It ensures the compatibility of the geometry description with the prevailing standard in Computer Aided Design [1,2]. Additional advantages include higher rates of convergence and increased stability of the simulation results. These beneficial effects are due to the increased smoothness and the higher polynomial degrees of the functions used to represent the simulated phenomena.

However, a NURBS representation of the computational domain, which is often the volume of a solid object or the volume surrounding a solid object, is not provided by a typical CAD model. In connection with the advent of isogeometric analysis, several authors presented algorithms for *creating a NURBS volume representation from a given CAD model*:

– Martin et al. [3] describe a method to generate a trivariate *B*-spline representation from a tetrahedral mesh. First, a volumetric parametrization of the genus-0 input mesh by means of discrete harmonic functions is constructed. This initial parametrization is then used to perform a *B*-spline volume fitting to obtain a *B*-spline representation of a generalized cylinder. An extension of this work to more general objects (e.g. to a genus-1 propeller) is presented in [4].
– Another parametrization method for a generalized cylinder-type volume is proposed in [5]. A NURBS parametrization of a swept volume is generated by using a least-squares approach with several penalty terms for controlling the shape of the desired parametrization. Among other applications, this method can be used to generate volume parametrizations for blades of turbines and propellers.
– Xu et al. [6] present a volume parametrization technique for a multi-block object. The parametrization of a single block is constructed by minimizing a quadratic objective function subject to two constraints. While one condition ensures the injectivity of

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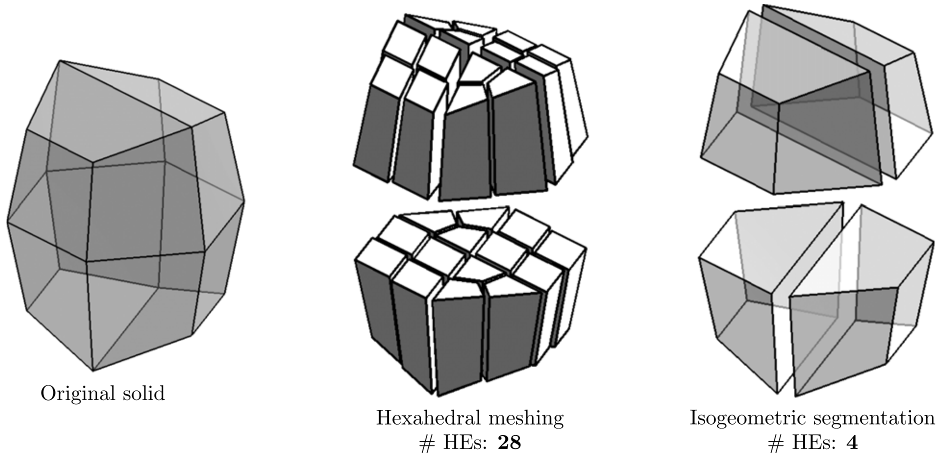

Fig. 1. An example that helps to distinguish our isogeometric segmentation (IGS) problem from the traditional hex meshing (THM) problem. *Left*: An example solid with a 3-vertex-connected edge graph. *Middle*: A natural decomposition of the example solid as a solution to the THM problem; we believe that this is the coarsest decomposition that does not possess T-joints. *Right*: A segmentation of the example solid as a solution to our IGS problem; we note that the segmentation admits T-joints.

the single *B*-spline parametrizations, the other condition guarantees C^1-smoothness between the blocks.
- Another volume parametrization method [7] generates first a mapping from the computational domain, which is given by its boundary, to the parameter domain by means of a sequence of harmonic maps. The parametrization of the computational domain is then obtained by a *B*-spline approximation of the inverse mapping.
- Given a boundary representation of a solid as a *T*-spline surface, which is assumed to have genus zero and to contain exactly eight extraordinary nodes, the algorithm in [8] constructs a solid *T*-spline parametrization of the volume.
- Further approaches to volume parametrization are described in [9–12].

Since many of the existing methods for (NURBS) volume parametrization are restricted to simple objects (cf. [6]) or to decompositions of more complex objects into simple ones, an algorithm for splitting a solid represented by a CAD model into a collection of simpler solids is of interest. In particular, *decompositions into solids that are topologically equivalent to hexahedra or tetrahedra* are desirable, since these objects can be easily parametrized by tensor-product NURBS volume patches.

- The decomposition of a convex polyhedron into a collection of tetrahedra is a well-studied problem [13,14]. A tetrahedralization of a convex polyhedron can be also generated by barycentric subdivision (cf. [15]), which can be applied to any connected polyhedral complex, see [16].
- For general polyhedra, several methods for decomposing them into smaller convex polyhedra have been studied, e.g. [17–19]. In contrast to the convex case [13], it is not always possible to obtain a tetrahedralization without adding new vertices, cf. [17].
- A well-established approach to the decomposition of a CAD model is the use of the geometric information that is provided by its features (e.g. sharp edges). Chan et al. [20] describe a volume segmentation algorithm that can be used for prototyping applications. The initial solid is repeatedly decomposed into smaller ones until all resulting models belong to a class of so-called "producible" solid components. It is ensured that the union of the constructed solids represents again the initial object.
- Other feature-based methods that have been described in the literature, (e.g. [21,22]) decompose polyhedral objects and special curved objects (i.e. objects with planar and cylindrical surfaces) into maximal volumes. In the method described in [21], the maximal volumes are always convex objects, whereas in [22] in some cases the maximal volumes may include objects with a few non-convex edges, too.
- Another approach to the segmentation of a CAD model is the representation as a hexahedral mesh with many hexahedra of approximately uniform size and shape. This is usually referred to as the problem of hex(ahedral) mesh generation. Due to the importance of hex meshes for numerical simulation, this problem has continuously attracted attention over the years. A feature-based algorithm for generating such meshes was introduced in [23], consisting of the following steps. The first phase is devoted to the feature recognition, which provides a guiding frame for the decomposition of the CAD model. Secondly, cutting surfaces are constructed, which split the initial solid into hex-meshable volumes. Further instances of hexahedral meshing algorithms are described in [24–30].

In contrast to these approaches, our goal is the decomposition of a CAD-model into a *small* number of topological hexahedra, which can be parametrized by single trivariate tensor-product NURBS-patches. More precisely, we consider the following *Isogeometric Segmentation Problem:*

> Given a solid object $\mathcal{S}$ (represented as a CAD model), find a collection of mutually disjoint topological hexahedra $\mathcal{H}_i$ ($i = 1, \ldots, n$) whose union represents $\mathcal{S}$. The shape of the topological hexahedra need not to be uniform, and the hexahedra are not required to meet face-to-face, thereby allowing T-joints. However, the number n of topological hexahedra should be relatively small.

Each of the topological hexahedra can be represented as a trivariate NURBS volume, which can then be used for performing a numerical simulation using the isogeometric approach. By using a small number of topological hexahedra, it is possible to exploit the regular tensor-product structure on each of them. Since the individual NURBS volumes may not meet face-to-face, advanced techniques (e.g., based on discontinuous Galerkin discretizations) for coupling the isogeometric discretization are required. In the context of IGA, such techniques are currently being investigated in [31].

We expect that a valid solution to the isogeometric segmentation problem can be obtained using a smaller number of topological hexahedra than the traditional hexahedral meshing. This is because

- the individual hexahedral elements do not need to meet face-to-face (cf. Fig. 1), and