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Topology-driven shape chartification

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A B S T R A C T

We propose a novel algorithm to decompose a 3D object into an atlas of disk-like charts. Decomposition into charts with controlled shape and topology is relevant in many engineering areas, such as spline fitting, compression and re-meshing. We produce our chartifications by jointly exploiting the Reeb graph of a guiding function and its gradient aligned flow paths. The key advancements of our method with respect to similar approaches are: (i) a novel strategy to provably remove all T-junctions; (ii) a stable system to trace flow paths starting far from critical points; (iii) the exploitation of the regularity of certain functions under isometries (e.g., harmonic ones) to produce structurally equivalent chartifications for families of objects posed differently. The charts produced by our system can be of two types: topological quads and topological octagons. Both of them can be easily gridded to produce full quadrilateral meshes, as we demonstrate in the second part of the article.

1. Introduction

Shape chartification is the process of partitioning an arbitrary surface into a set of charts having simpler topology and geometry (Zhang et al., 2005). As demonstrated by recent research in the field (Xiao et al., 2018; Buchegger and Jüttler, 2017; Hu et al., 2017; Hu and Zhang, 2016), in the CAD/CAE community it is often convenient to have charts that can be easily gridded, and also to make sure that the chartification does not contain T-junctions (Myles et al., 2010). Furthermore, chartifications are beneficial in a whole variety of applications, including remeshing (Patanè et al., 2004; Pietroni et al., 2010; Tarini et al., 2011; Cherchi et al., 2016), spline fitting (Campen and Zorin, 2017), texturing (Purnomo et al., 2004; Usai et al., 2015), compression (Choe et al., 2009), shape approximation (Cohen-Steiner et al., 2004) and fabrication (Julius et al., 2005).

Our goal is to produce a chartification that remains consistent if the surface undergoes deformations that, at least to some extent, preserve geodesic distances (i.e. isometries). Moreover, we target a chartification that is T-junction free and keeps the valence of the chart vertices as regular as possible. The solution presented in this paper adopts a topology-driven approach whose ingredients are Reeb graphs and discrete gradient flow paths. The core novelty relies in the way they are combined together to produce the final decomposition.

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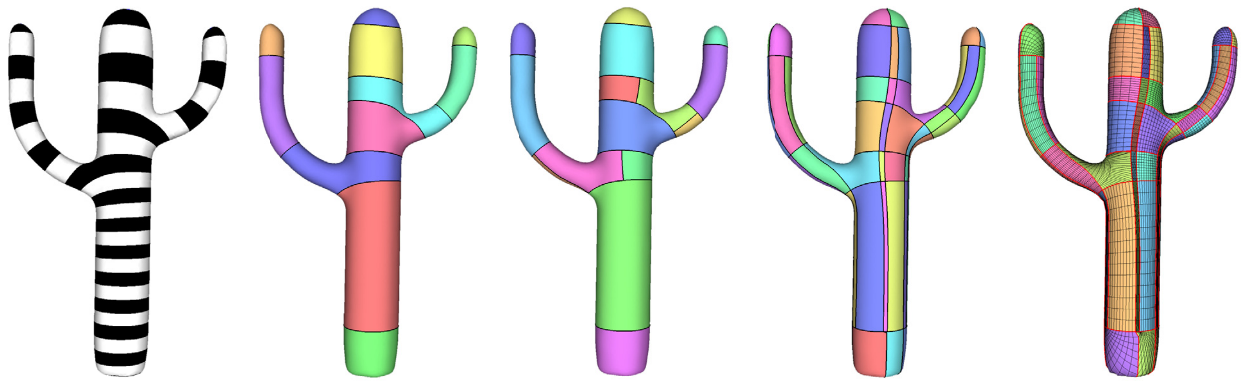

Fig. 1. Our chartification algorithm in a nutshell. Left: we start from a triangle mesh and a guiding function; middle left: we extract the Reeb regions (caps, saddles, cylinders); middle: we refine saddle charts, creating rhombus domains; middle right: we propagate and eventually delete the T-junctions generated at the previous step; right: we grid each domain to produce a quadrilateral remeshing of the input shape.

The Reeb graph of a guiding scalar field is used to extract a coarse chartification of the surface, isolating cylindrical and saddle areas, and caps. These initial charts are then refined by tracing piece-wise linear curves aligned with the function gradient, ensuring the elimination of all T-junctions and the construction of only 4-sided and *rhombus* charts. Rombus charts are 8-sided charts enclosing saddles. The strategy adopted for refining the coarse Reeb atlas distributes the boundaries of the charts evenly with respect to the behavior of the field over the surface, and sufficiently far away from the critical points to avoid instability of the chart boundary positioning, a known critical issue in previous similar approaches (Dong et al., 2005; Huang et al., 2008). An overview of the chartification approach is shown in Fig. 1.

While this machinery is agnostic to the function being used and could potentially be adopted for any function which admits a Reeb Graph (i.e., Morse–Smale), the choice of functions which are invariant under isometries is the key to ensure consistent chartifications across different poses of the same shape. There is practical evidence that harmonic functions exhibit a consistent behavior when boundary conditions are well placed (Bærentzen et al., 2014) (e.g., at the extrema of protuberances). In our experiments we mostly rely on harmonic functions, producing consistent decompositions for shapes belonging to the same class. For completeness, in Section 6 we show a few chartifications obtained with alternative functions.

Methods that adopt similar chartification approaches have been presented in literature, but either they are restricted to a small class of shapes (e.g., tubular Usai et al., 2015) or suffer from presence of T-junctions or instability in the quad layout (see Section 2 for a detailed discussion). The main achievements of our method with respect to existing ones can be summarized as follows:

- *consistency across deformations*: the chartification is primarily guided by the topological structure induced by the critical points of a harmonic field defined over the surface, whose critical points are well behaving with respect to deformations of the surface and therefore ensuring consistency of the topological structure across isometric deformations (Fig. 1);
- *stability of the chart structure*: we propose a method to trace separatrices of our coarse Reeb charts as flow paths of the underlying scalar function. The key point of our tracing system is that we start tracing them far from critical points (Fig. 1(a)–(b)). Other approaches, for instance Dong et al. (2005), Huang et al. (2008), trace separatrices starting from saddle points. We observe that this may lead to unstable behavior, because the gradient is not defined in critical points and, due to discretization issues, is also usually unstable nearby;
- *T-junction free chartification*: our surface charts are topological quads with no T-junctions. The most recent approach that is able to obtain such a result Tierny et al. (2012) deletes T-junctions by using a geometric greedy stitching algorithm that is not always able to remove all of them. Our T-junction removal system is more general and robust, and always guarantees a T-junction free chartification (Fig. 1(c)).

We demonstrate our chartification algorithm in the context of quadrilateral remeshing (Fig. 1(d)). In Section 5 we discuss a simple yet effective method to map both 4- and 8-sided charts onto proper quadrangular domains, producing a quadrilateral tessellation of the input surface. By exploiting the regularity of harmonic functions we also show how to generate consistent quadrilateral meshes of similar shapes having different discretization and no cross-parameterization. Differently from the recently published method (Azencot et al., 2017) which produces quad meshes with *similar* structure, our topological approach produces *exactly* the same structure (same number of singular vertices and separatrices).