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A new variant of Lane–Riesenfeld algorithm with two tension parameters ☆

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ABSTRACT

We present a generalization of Lane–Riesenfeld algorithm with two tension parameters for curve design. The generalization incorporates existing families as special cases: Hormann–Sabin's family, Romani's family, J-spline family, and Siddiqi's improved binary four-point family. The new family also has common members with dual de Rham-type approximating schemes when n is even. Analysis of the new family indicates that there are parameter choices that provide higher continuity and higher generation degree than available with any of those existing families.

1. Introduction

Subdivision is a simple and popular method in geometric modeling. Given a control polygon, subdivision can generate smooth curves by recursively refining the polygon according to certain refinement rules. The Lane–Riesenfeld algorithm (Lane and Riesenfeld, 1980) (LR algorithm for short) provided a subdivision scheme for generating arbitrary-degree uniform B-spline curves. With the efficient local averaging rules, LR algorithm attracted a lot of attention and many variants have been proposed to meet different needs in geometric design. For example, Schaefer et al. (2008) suggested using a non-linear averaging rule to replace the linear smoothing operator to obtain schemes for non-polynomial functions, e.g. exponential functions. Hormann and Sabin (2008) proposed a new family of subdivision schemes with similar structure to that of B-spline schemes but having cubic precision. Schaefer and Goldman (2009) provided an efficient algorithm for subdividing non-uniform B-splines in a way like LR algorithm for uniform B-splines. Cashman et al. (2013) generalized LR algorithm by replacing the linear averaging rule with higher-order local interpolation operator both for the refine and the successive smoothing stages to reproduce cubic polynomials, circles, and so on. Romani (2015) proposed a variant of LR algorithm with smoothing stages unchanged but the refine stage based on a parameter-dependent variant of Chaikin's algorithm (Chaikin, 1974), and many well-known subdivision schemes are special cases of this variant.

In this paper, we present a new variant of LR algorithm with the refine stage having two tension parameters, which is also a generalization of Romani's schemes. The new variant has three subfamilies which are not contained in the Romani's family. Two are known subdivision schemes, and one is a generalization of Hormann–Sabin's family. Analysis of the new family indicates that there are parameter choices that provide higher continuity and higher generation degree than available with any of those existing families.

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This paper is organized as follows. In Section 2, we recall the LR algorithm. In Section 3, a new variant of LR algorithm with the refine stage having two tension parameters is proposed. The properties of the variant are studied in Section 4. In Section 5, we give some examples. Conclusions are given in Section 6.

2. LR algorithm

LR algorithm provided a unified framework to subdivide arbitrary-degree uniform B-spline curves. In the algorithm, each subdivision step T contains two parts: one refine stage R_L and n smoothing stages S_L. Given a set of initial control points $\boldsymbol{P}=\{P_i\}_{i\in\mathbb{Z}}$, the refine stage is:

$$\begin{cases} (R_L\boldsymbol{P})_{2i}=P_i \\ (R_L\boldsymbol{P})_{2i+1}=\frac{1}{2}P_i+\frac{1}{2}P_{i+1} \end{cases} \tag{1}$$

the smoothing stage is:

$$(S_L\boldsymbol{P})_i=\frac{1}{2}P_i+\frac{1}{2}P_{i+1} \tag{2}$$

and $T=S_L^n\cdot R_L$. With n increased, higher degree splines can be got.

In the literature, processes containing the refine stage and the smoothing stage are called Refine-and-Smooth algorithms, and LR algorithm may be the simplest one, since its averaging rules in refine operator R_L and smoothing operator S_L are both local linear interpolants. Changing the refine operator or the smoothing operator can get different variants of LR algorithm and meet different designing needs. In section 3, we will present a new variant with tension parameters which only modifies the refine operator.

3. Construction of the new variant

Romani (2015) proposed a Chaikin-based variant of LR algorithm $S_{a_{n,w}}$ with symbol

$$a_{n,w}(z)=\left(\frac{1+z}{2}\right)^{n+1}\cdot\left(-(n+3)w+8wz+2(1+(n-5)w)z^2+8wz^3-(n+3)wz^4\right), \tag{3}$$

which is actually a unified subdivision scheme that contains primal and dual schemes with tension parameter. Inspired by this result, we generalize $S_{a_{n,w}}$ by changing the refine operator and obtain a new variant of LR algorithm $S_{n,s,t}$ $(n\in\mathbb{N})$ which provides a larger unified frame containing more primal and dual schemes with tension parameter. The new family of subdivision schemes $S_{n,s,t}$ has the symbol:

$$a_{n,s,t}(z)=\frac{(1+z)^{n+1}}{2^n}\cdot b_{s,t}(z), \tag{4}$$

where

$$b_{s,t}(z)=s(1+z^4)+t(z+z^3)+(1-2s-2t)z^2.$$

Here, $n\in\mathbb{N}$ is the number of smoothing stages, and is also used to identify the family member. Denote the refine stage by $R_{s,t}$, then the explicit form of $R_{s,t}$ is:

$$\begin{cases} P_{2i}^{k,1}=(s+t)P_{i-1}^{k,0}+(1-2s-t)P_i^{k,0}+sP_{i+1}^{k,0}, \\ P_{2i+1}^{k,1}=sP_{i-1}^{k,0}+(1-2s-t)P_i^{k,0}+(s+t)P_{i+1}^{k,0}. \end{cases} \tag{5}$$

Below we will list some special members of $S_{n,s,t}$.

3.1. B-spline refinement

The symbol of $S_{n,0,0}$ is

$$\frac{(1+z)^{n+1}}{2^n}\cdot z^2,$$

which is obviously the symbol of degree-n uniform B-spline refinement.