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New algorithm to find isoptic surfaces of polyhedral meshes

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The isoptic surface of a three-dimensional shape is recently defined by Csima and Szirmai (2016) as the generalization of the well-known notion of isoptics of curves. In that paper, an algorithm has also been presented to determine isoptic surfaces of convex polyhedra. However, the computation of isoptic surfaces by that algorithm requires extending computational time and CAS resources (in Csima and Szirmai, 2016 *Wolfram Mathematica* Inc, 2015 was used), even for simple regular polyhedra. Moreover, the method cannot be extended to concave shapes. In this paper, we present a new searching algorithm to find points of the isoptic surface of a triangulated model in $\mathbb{E}^3$, which works for convex and concave polyhedral meshes as well. Alternative definition of the isoptic surface of a shape is also presented, and isoptic surfaces are computed based on this new approach as well.

1. Introduction

In CAGD systems there are only primitive tools available to determine camera positions to display and study three-dimensional objects from various directions. However, in certain applications, it can be important to find positions, from where a predefined portion of a given object can be seen.

In two dimensions we can use viewing angle to determine such points which led us to the well-known definition of isoptic curves of a given curve. For arbitrary given plane curve, the isoptic curve is the locus of those points of the plane from where the given curve can be seen under a predefined angle (of less than π). These points can be determined by drawing tangents to the given curve—the tangents have to meet at the given angle in these points (if there are more than two tangents from a point to the curve, then the widest angle is considered). Isoptics of several classical curves have been studied from the ancient Greek time to present (see, e.g. Yates, 1947 for an overview of these results).

In terms of recent applications, appropriate camera points or paths can be determined using isoptics. Although theoretical results can also be of great interest, the ultimate goal of the present paper is to provide a fast computation of isoptics that can be used as camera positions in a modeling software.

Extension of the two-dimensional principle to three-dimensional shapes is not trivial and not unique. One can try to find isoptic curves of the spatial object in a given "base" plane. Our previous work was such an extension of the isoptic curves to $\mathbb{E}^3$, described in Nagy and Kunkli (2013), together with a computation of the isoptic curve of a Bézier surface in a special case, that can be used as a camera path around the given Bézier surface (see Fig. 1).

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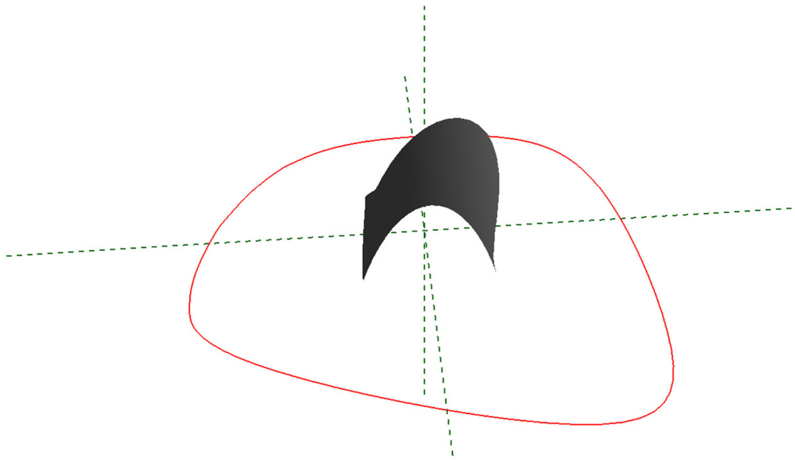

Fig. 1. Isoptic curve of a Bézier surface on a fixed base plane with $\frac{1}{2}\pi$ (red). (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

If we consider isoptics as hypersurfaces of the given space, then one can also try to define and compute an isoptic surface in 3D, but then the given angle of the planar case must be substituted by an appropriate notion and measure. Bearing the applications in mind, we can consider this angle as the measure of visibility. In case of curves, it can also be measured by the assigned arc length of a unit circle around the viewpoint. Then the obvious generalization of this notion is the area of visibility in a unit sphere around the viewpoint, which was precisely defined in Csima and Szirmai (2013) and will be discussed in Section 2, together with the brief overview of the existing method. Unfortunately, that method works only for convex shapes and takes several minutes to calculate the isoptic surface. Our aim is to provide a fast and robust method to compute the isoptic surface, even for concave meshes. This new algorithm is presented in Section 3.

One can also generalize the 2D notion in an alternative way, by considering the isoptic point in the plane as the intersection of two tangent lines to the curve, i.e. the "widest view" of the curve from that point. With this approach, the 3D generalization of the angle can be the widest spatial view, that is the measure of the widest diameter of the projection of the shape onto the unit sphere around the viewpoint. This generalization will be briefly discussed in Section 4.

2. Previous results

In this section, we briefly summarize the earlier approaches and recall the notion of 3D isoptics as an extension of the 2D measure of angles. This extension has been given by Csima and Szirmai (2013) based on the following principle: in the curve case the angle (of view) in vertex A on the plane can be measured by the arc length on the unit circle around the point A. This angle can be generalized to the Euclidean space by the definition of the solid angle (Gardner and Verghese, 1971) (see Fig. 1 in Csima and Szirmai, 2013):

Definition 1. The solid angle $\Omega_{\mathcal{S}}(P)$ subtended by a surface $\mathcal{S}$ is defined as the surface area of the projection of $\mathcal{S}$ onto the unit sphere around P.

Based on this notion the isoptic hypersurface is defined as follows (Csima and Szirmai, 2016):

Definition 2. The isoptic hypersurface $\mathcal{H}^{\alpha}_{\mathcal{D}}$ in $\mathbb{E}^3$ of an arbitrary 3-dimensional compact domain $\mathcal{D}$ is the locus of points P where the measure of the projection of $\mathcal{D}$ onto the unit sphere around P is equal to a given fixed solid angle value α, where $0<\alpha<2\pi$ (see, e.g. Fig. 2).

Due to the fact that the surface area of a unit sphere is 4π, the natural upper bound for the solid angle value α is 2π in case of convex shapes, because the projection of a convex shape onto a unit sphere cannot cover more than a half sphere. It does not hold for concave meshes, where the area of projection can be larger than the half sphere.

A method to determine the isoptic surface of convex polyhedra is also provided in Csima and Szirmai (2016). If we assume that an arbitrary convex polyhedron is given by the list of facets and the set of vertices, the area of a projected facet can be calculated as the area of the spherical polygon that the facet covers on the unit sphere. Then the solid angle of the polyhedron is the half of the summarized area of all the projected facets. By solving the mathematical formula, this algorithm provides the implicit equation of the compound isoptic surface of the given convex polyhedron.

However, these computations are extremely complicated even for simple regular polyhedra. Therefore, it is possible to solve and plot it only by computer algebra systems, and it takes around 20–40 minutes to display the isoptic surface. Moreover, as it has already mentioned, the method presented by Csima and Szirmai works only for convex meshes.

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