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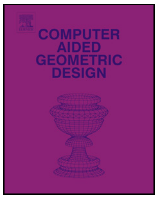

# Accelerated robust Boolean operations based on hybrid representations

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## ABSTRACT

Constructive Solid Geometry (CSG) is one of the popular techniques that is widely applied in 3D modeling. It combines primitive solids using Boolean operations. However, the trade-off between efficiency and robustness of Boolean evaluation is difficult to balance. Previous methods sacrifice either efficiency or robustness to achieve advantages in one perspective. Recent works attempt to achieve excellent performance in both aspects through replacing the conventional vertex-based representations (V-reps) with plane-based representations (P-reps) of polyhedrons. Different from V-reps, the P-reps use plane coefficients as meta-data and can lead to benign robustness. However, methods using P-reps have disadvantages in efficiency compared to methods using V-reps. In this paper, we proposed a Boolean evaluation approach that absorbs both the efficiency of V-reps based methods and robustness of P-reps based methods. We design a Boolean evaluation method combining P-reps with V-reps. The P-reps information is utilized for exact predicate computation while information in V-reps is collected for fast topology query and coarse tests. Our proposed approach is variadic: it evaluates a Boolean expression regarding multi-input meshes as a whole rather than a tree of decomposed binary operations. We conduct massive experiments and compare our results with those generated by the state-of-the-art methods. Experimental results show that our approach is robust for solid inputs and has advantages in performance compared to some previous non-robust methods.

## 1. Introduction

Constructive Solid Geometry (CSG) is a popular modeling technique for Computer-Aided Design (CAD) and Computer-Aided Manufacturing (CAM). Through using regularized Boolean operations (union, intersection and difference) (Requicha, 1977; Tilove and Requicha, 1980), complex models can be easily constructed with combined primitives. There are mainly two categories of Boolean evaluation methods which are different in processing the intersections between primitives. One

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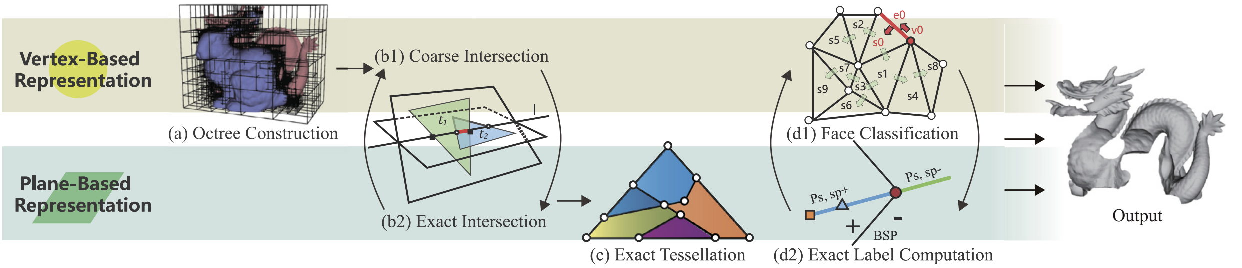

**Fig. 1.** An overview of our Boolean evaluation framework based on hydrid representations. Vertex-based computation is included to maintained high efficiency and plane-based computation is conducted to ensure exact geometry predicates. Our approach mainly contains three stages: intersection computation, face tessellation and face classification.

category is the *approximate method* (Pavić et al., 2010; Biermann et al., 2001; Wang, 2011). These methods approximately fit the vertexes after discretizing the intersection areas, then rearrange the topology. The other category is the *exact method* (Douze et al., 2015; Ogáyar-Anguita et al., 2015; Zhou et al., 2016). These methods are characterized by preservation of the vertex positions and maximum maintenance of multiple input elements (such as faces, vertexes, and topology). Many applications apply exact methods instead of approximate methods to pursue better accuracy. In addition, exact methods provide an accurate mapping between the input surfaces and the output meshes. This map benefits the transfer of surface information such as face colors and materials. We focus on the stream of exact methods in this paper.

There exists a trade-off between *robustness* and *efficiency* when designing Boolean algorithms. The robustness of the methods is usually guaranteed by exact arithmetic (Barki et al., 2015; Zhou et al., 2016), which is significantly slower than methods using normal floating-point arithmetic. Some methods apply other techniques such as epsilon-tweaking (Laidlaw et al., 1986; Segal, 1990; Feito et al., 2013) and numerical perturbation (Douze et al., 2015) but can only achieve **quasi-robustness** (Shewchuk, 1999). One important feature of these quasi-robust methods is the polyhedron representations based on vertexes (V-reps). However, V-reps is not the only choice. Sugihara and Iri (1989) proposed the plane-based representation (P-reps) for polyhedrons. The advantages of applying P-reps as primary geometric information is that the rudimentary modeling operations can be conducted robustly. Under P-reps, the evaluation of Boolean expressions is free from constructing new primary geometry information. In other words, no **constructions** are needed when applying P-reps and the computations are restricted to **predicates** only. Some researchers (Bernstein and Fussell, 2009; Campen and Kobbelt, 2010) picked up the P-reps and coupled them with Binary Space Partitioning (BSP) structures to develop an exact and robust Boolean operator. Although these plane-based methods are generally faster than those relying on exact arithmetic (Hachenberger and Kettner, 2005, 2006; Granados et al., 2003), plane-based methods are still limited by the high computational complexity of the BSP algorithms. Additionally, to make the boundary representations of polyhedrons compatible with the BSP structure, extra steps of conversion and connectivity reconstruction are inevitable and further deteriorates the runtime performance.

To properly handle the trade-off between efficiency and robustness, we develop a robust approach for Boolean evaluation, which has sound robustness with consistent solid inputs without sacrificing efficiency. To achieve excellent performance both in robustness and efficiency, we design a hybrid representation of solids combining P-reps with V-reps as shown in Fig. 1. It absorbs the advantages of high efficiency of methods based on V-reps and strong robustness of methods using P-reps simultaneously. In our approach, we take advantages of the V-reps information for coarse intersection and efficient neighboring face queries in the face classification process. The P-reps information is used for exact geometry computations. Our approach successfully increases the efficiency by avoiding constructions with exact arithmetic.

The architecture of our approach is more similar to vertex-based methods (Feito et al., 2013; Zhou et al., 2016) than BSP-based methods (Bernstein and Fussell, 2009; Campen and Kobbelt, 2010). Note that our approach is a systematic solution of robust Boolean evaluations rather than a simple improved version of the robust vertex-based method. Our approach mainly contains three stages: intersection detection, face tessellation and face classification. In the first stage, triangle intersections are encoded into sets of planes, which are used for determining exact tessellations (see Fig. 1 (b1) & (b2)). Subsequently, faces in the tessellated meshes are classified using small local BSP trees, which are also compatible to the P-reps for ensuring exactness (Fig. 1 (d1) & (d2)). Besides the feature of high efficiency and robustness, our method is also **variadic** (Zhou et al., 2016), which means our approach is able to evaluate the whole input meshes without decomposition. These features save a large amount of evaluation time by avoiding repetitive computation. In sum, our approach has the following contributions:

- **Plane-based intersection test:** We develop a triangle–triangle intersection test suitable for geometric objects in plane-based representation.
- **Deferred tessellation using Tess-Graph:** The tessellations of faces are conducted after all the intersections are detected and refined. To avoid errors caused by extra conversion, we develop a Tess-Graph to extract subfaces.

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