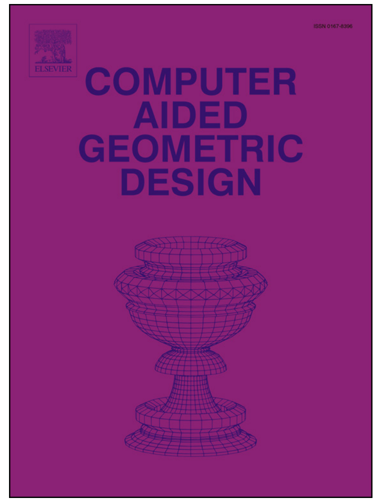
Accepted Manuscript

Multisided generalisations of Gregory patches

Gerben J. Hettinga, Jiří Kosinka



PII: S0167-8396(18)30019-0
DOI: https://doi.org/10.1016/j.cagd.2018.03.005
Reference: COMAID 1662

To appear in: *Computer Aided Geometric Design*

Please cite this article in press as: Hettinga, G.J., Kosinka, J. Multisided generalisations of Gregory patches. *Comput. Aided Geom. Des.* (2018), https://doi.org/10.1016/j.cagd.2018.03.005

Multisided Generalisations of Gregory Patches

Gerben J. Hettinga[1], Jiří Kosinka[1]

[1] Johann Bernoulli Institute, University of Groningen, 9747 AG Groningen, The Netherlands

Abstract

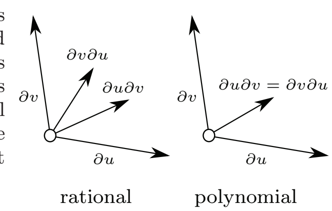
We propose two generalisations of Gregory patches to faces of any valency by using generalised barycentric coordinates in combination with two kinds of multisided Bézier patches. Our first construction builds on S-patches to generalise triangular Gregory patches. The local construction of Chiyokura and Kimura providing G^1 continuity between adjoining Bézier patches is generalised so that the novel Gregory S-patches of any valency can be smoothly joined to one another. Our second construction makes a minor adjustment to the generalised Bézier patch structure to allow for cross-boundary derivatives to be defined independently per side. We show that the corresponding blending functions have the inherent ability to blend ribbon data much like the rational blending functions of Gregory patches. Both constructions take as input a polygonal mesh with vertex normals and provide G^1 surfaces interpolating the input vertices and normals. Due to the full locality of the methods, they are well suited for geometric modelling as well as computer graphics applications relying on hardware tessellation.

1 Introduction

The creation of G^1 (or tangent plane) continuous surfaces from a polygonal mesh has long been a problem in computer graphics and geometric modelling. Attaining this level of continuity locally, by using information from only one polygon at a time, is a difficult problem. By augmenting the positional data of the mesh with vertex normals it becomes possible to assure G^1 continuity between adjacent patches by taking into account only locally defined information. Complex smooth shapes are easily modelled by automatically generating patches based on coarse polygonal meshes. In addition, such techniques are well suited for applications in computer graphics relying on hardware tessellation.

This work describes the augmentation of the S-patch structure [20] with Gregory terms and rational blending functions. Moreover, the method of Chiyokura and Kimura [4] is generalised to multisided faces such that a Gregory S-patch can be joined smoothly to adjacent (Gregory or Bézier) patches. The combination of these results in a multisided generalisation of triangular Gregory patches. In addition, the method of Chiyokura and Kimura is also applied to generalised Bézier patches [28]. By adjusting the control point structure of this patch it becomes possible to define cross-boundary derivatives independently per side. This effectively creates a multisided generalisation of quadrilateral Gregory patches.

G^1 continuity is a less strict version of the parametric C^1 continuity. Essentially, it means that along the shared boundary of two parametric surfaces the surface normal field is shared as well. When considering the problem of joining patches with G^1 continuity, some difficulties arise. It is often the case when using polynomial constructions that cross-boundary derivatives can be matched on each one of the boundary edges separately, but internally the mixed partial derivatives, $\partial u \partial v$ and $\partial v \partial u$, cannot. This is known as the vertex inconsistency problem or the twist compatibility problem. A commonly used solution is to employ rational functions that blend the derivatives accordingly.



Multisided surface patches are well-established in computer graphics and geometric modelling. They offer the ability to smoothly interpolate data over polygonal domains such that visually pleasing surface patches are created. However, the problem of how to smoothly join several of these patches together has not been given much attention. One such multisided patch is the S-patch [20], which is a multisided generalisation of ordinary Bézier patches. In Section 3 the conditions of G^1 continuity for S-patches are reviewed and it is shown how Chiyokura and Kimura's method can be adapted to S-patches. We first look at quadrilaterals (Section 3.1) and then show how the structure generalises to arbitrary polygons (Section 3.2).

Another representation of multisided surfaces was developed by Varady et al. [28]. This multisided patch, called the generalised Bézier patch, has a very compact structure compared to S-patches and combines transfinite and control point oriented structures. Section 4.1 shows how the structure of the generalised Bézier patch can be adjusted such that it blends ribbons in a Gregory-like manner and guarantees G^1 continuity between adjacent patches.

We then present and discuss results obtained by the two methods (Section 5) and conclude the paper (Section 6). But before all that, we review existing work and recall relevant constructions ours build upon in the next section.

2 Related Work

There are several existing techniques that locally create C^0 continuous triangular patches from meshes with normals. Phong tessellation [2] and PN triangles [30] provide quadratic and cubic Bézier patches, respectively. Recently, both schemes have been generalised to arbitrary n-sided polygons by use of generalised barycentric coordinates [13]. The patches can be constructed locally per polygon and are therefore suitable for hardware tessellation [24].

Locally constructing C^1 patches is a well studied problem, particularly in the finite element literature. For example, Clough and Tocher [5] create smooth piece-wise cubic interpolants over a triangulated domain based on gradients at vertices and cross-boundary derivatives at midpoints of edges. By splitting the original domain triangle into three micro-triangles, C^1 continuous surfaces are constructed. Gregory et al. [10] modified a bi-cubic Coons patch such that derivatives at vertices can be specified without having to adhere to compatibility conditions. Mixed partial derivatives are blended by rational blending functions such that C^1 continuity is achieved. However, achieving C^1 continuity is not possible for arbitrary polygonal meshes due to topological constraints imposed by global parametrisations. Therefore, G^1 continuity is often more appealing.

Chiyokura and Kimura [4] created a special Bézier patch, similar to Gregory's patch, where cross-boundary tangents can be defined independently per boundary edge. Through the use of rational blending functions, edge conditions are blended smoothly